Observer synthesis for linear discrete-time systems with different fractional orders

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Abstract: The paper is devoted to observer synthesis for linear discrete-time positive fractional systems with different fractional orders. The problem of finding a nonnegative gain matrix of the observer such that the observer is positive and asymptotically stable is formulated and solved by the use of linear programming (LP) and linear matrix inequality (LMI) methods. The proposed approach to the observer synthesis is illustrated by theoretical example. Numerical calculations and simulations have been performed in the MATLAB/Simulink program environment.

Keywords: fractional, positive, discrete-time, system, observer, linear programming, linear matrix inequality

Many sophisticated analytical procedures to control system design are based on the assumption that the full state vector of the system is available for measurement. The example of such control procedure is placement of the unstable system eigenvalues. In many practical systems the entire state vector is not available for measurement. In some cases measurements may require the use of costly measurement devices and it may be unreasonable to measure all state variables. An auxiliary dynamical system, which reconstructs the state vector, is known as a full-order or an identity observer, and is coupled to the original system through the available system inputs and outputs [1].

In this paper the positive fractional discrete-time systems will be considered. In positive dynamical systems each inputs, state variables and outputs take only non-negative values. Examples of such systems are processes involving chemical reactors, distillation column, compartmental systems or atmospheric pollution models [2]. Dynamical systems described by fractional order differential or difference equations have been investigated in several areas such as viscoelasticity, diffusion processes, electrochemistry, control theory, electrical engineering, etc. (see [3–6] and references therein, for example).

The problem of the observer synthesis (full-order and reduced-order) for fractional discrete-time systems have been studied for example in [7, 8]. An linear matrix inequality (LMI) approach to observer synthesis for positive discrete-time integer order systems has been proposed in [9] and linear programming (LP) approach in [10].

The considerations presented in this paper are the complement of the general control theory of the fractional discrete-time systems and can be applied in different areas of sciences.

1. Problem formulation

Let us denote by $\Re^{n\times m}$ ($C^{m\times n}$) the set of real (complex) matrices with n rows and m columns and $\Re^n = \Re^{n\times 1}$. The set of real $n\times m$ matrices with nonnegative entries will be denoted by $\Re_+^{n\times m}$ ($\Re_+^n = \Re^{n\times 1}$). A matrix $A=[a_{ij}]\in\Re_+^{n\times m}$ (a vector $x=[x_i]\in\Re_+^n$) will be called strictly positive and denoted by $A>0$ if $a_{ij}>0$, $i=1,2,..n$; $j=1,2,\ldots,m$ (by $x>0$ if $x_i>0$, $i=1,2,..n$). The set of nonnegative integers will be denoted by Z_+. The set of $n\times n$ symmetric matrices will be denoted by S^n. A matrix $Q\in S^n$ is positive (negative) define $Q\succ 0$ ($Q\prec 0$) if its quadratic form is positive (negative), i.e. $x^TQx>0$ ($x^TQx<0$) for every nonzero $x\in\Re^n$. The symbol "$\forall$" should be read "for all" and the symbol "$\in$" should be read "is an element of".

Let us consider the discrete-time fractional system of the form [8]:

$$\Delta^{\overline{\alpha}}x_{i+1} = Ax_i + Bu_i, \quad i\in Z_+,$$
$$x_{i+1} = \Delta^{\overline{\alpha}}x_{i+1} - \sum_{j=1}^{k+1}(-1)^j\overline{\alpha}x_{i+1-j}, \tag{1}$$

$$y_i = Cx_i, \tag{2}$$

with different orders ($\alpha_1\neq\alpha_2\neq\ldots\neq\alpha_n$) where $A\in\Re^{n\times n}$, $B\in\Re^{n\times m}$, $C\in\Re^{p\times n}$, $\alpha_r\in(0,1)$, $r=1,\ldots,n$, and

$$x_i=\begin{bmatrix}x_i^1\\ \vdots\\ x_i^n\end{bmatrix},\quad \Delta^{\overline{\alpha}}x_{i+1}=\begin{bmatrix}\Delta^{\alpha_1}x_i^1\\ \vdots\\ \Delta^{\alpha_n}x_{i+1}^n\end{bmatrix}, \alpha_r\in(0,1),\ r=1,\ldots,n$$
$$\overline{\alpha}_k = diag\left[\begin{pmatrix}\alpha_1\\ k\end{pmatrix} \ \ldots\ \begin{pmatrix}\alpha_n\\ k\end{pmatrix}\right]. \tag{3}$$

In (3) x_i, u_i, y_i are the state, input and output vectors.

The following conditions for the system (1), (2) can be proved in the same manner as for the positive fractional system with the same order α ($\alpha_1=\alpha_2=\ldots=\alpha_n$), see [13], for example.

Lemma 1. If

$$0<\alpha_l<1, \quad l=1,\ldots,n \tag{4}$$

then

$$(-1)^{i+1}\overline{\alpha}_k>0, \tag{5}$$

where $\overline{\alpha}_k$ has the form (3).

Lemma 2. If (4) holds and

$$(A + \overline{\alpha}_1) \in \Re_+^{n\times n} \tag{6}$$

then

$$\Phi_k^{\overline{\alpha}} \in \Re_+^{n\times n}, \quad k = 1,2,... \tag{7}$$

Theorem 1. The fractional system (1), (2) is positive (internally) if and only if

$$A_{\overline{\alpha}} = (A + \overline{\alpha}_1) \in \Re_+^{n\times n}, \ B \in \Re_+^{n\times m}, \ C \in \Re_+^{p\times n}. \tag{8}$$

In the next part of the paper, we will consider the fractional system (1), (2) with $\alpha_1 \neq \alpha_2 \neq ... \neq \alpha_n$ as the positive system (according with Theorem 1) with the scalar output y and $C = [c_1 \ \ ... \ \ c_n] \in \Re_+^{1\times n}$. We will assume that the input $(u_i \in \Re_+^m)$ and the output $(y_i \in \Re_+^p)$ variables of the system can be directly measured.

Definition 1. The state (full-order) observer of the system (1), (2) is the system which estimates the state variables $x_i \in \Re_+^n$ (3).

Definition 2. The observer of the system (1), (2) is given by the following equation:

$$\Delta^{\overline{\alpha}}\hat{x}_{i+1} = (A_{\overline{\alpha}} - LC)\hat{x}_i + Bu_i + Ly_i, \quad i \in Z_+,$$
$$\hat{x}_{i+1} + \sum_{j=1}^{k+1}(-1)^j \overline{\alpha}_j \hat{x}_{i+1-j} = (A_{\overline{\alpha}} - LC)\hat{x}_i + Bu_i + Ly_i. \tag{9}$$

where

$$\hat{x}_i = [\hat{x}_i^1 \quad ... \quad \hat{x}_i^n] \in \Re_+^n, \tag{10}$$

is an estimate of the state variable $x_i \in \Re_+^n$ and $L \in \Re_+^{n\times p}$ is a gain matrix of the observer.

From the equation (9) it follows that the observer for the positive system (1), (2) should be positive.

Definition 3. The set of all $\lambda \in \mathcal{C}$ which are the eigenvalues of $A \in \mathcal{C}^{n\times n}$ is called the spectrum of A and is denoted by $\sigma(A)$.

Definition 4. The matrix $A = [a_{ij}] \in \Re^{n\times n}$ is called a Schur matrix if it has all eigenvalues with moduli less than one, i.e. $|\lambda_i| < 1, i = 1,2,...,n$ where λ_i, $i = 1,2,...,n$ are the eigenvalues of A.

The main purpose of the paper is to give conditions for the existence of the observer (9) for discrete-time positive fractional system (1), (2) with different fractional orders ($\alpha_1 \neq \alpha_2 \neq ... \neq \alpha_n$) and a method for computation of the gain matrix $L \in \Re_+^{n\times p}$ of the asymptotic stable positive observer.

2. The main result

In this paragraph we shall show that the problem of observer synthesis can be reduced to a feasibility problem of: a linear programming (LP) and a linear matrix inequality (LMI).

Linear programming method

The linear programming (LP) is the problem of maximizing or minimizing a linear function over a convex polyhedron specified by linear and non-negativity constraints [14]. This problem can be expressed in canonical form:

$$\begin{aligned} &\text{maximize} && c^T x \\ &\text{subject to} && Ax \leq b \\ &\text{and} && x \geq 0 \end{aligned} \tag{11}$$

where x represents the vector of variables (to be determined), c and b are vectors of known coefficients. The expression to be maximized or minimized is called the objective function. The inequalities $Ax \leq b$ are the constraints which specify a convex polytope over which the objective function is to be optimized.

Let the vector of an error of the estimate has the form:

$$e_i = (x_i - \hat{x}_i) \in \Re_+^n, \quad i \in Z_+. \tag{12}$$

Substituting (1) and (9) into (12) we get:

$$e_{i+1} = x_{i+1} - \hat{x}_{i+1} = Ge_i - \sum_{j=1}^{k+1}(-1)^j \overline{\alpha}_j \hat{x}_{i+1-j},,$$
$$e_{i+1} = \Delta^{\overline{\alpha}} e_{i+1} - \sum_{j=1}^{k+1}(-1)^j \overline{\alpha}_j \hat{x}_{i+1-j}; \quad \Delta^{\overline{\alpha}} e_i = Ge_i \tag{13}$$

where

$$G = (A_{\overline{\alpha}} - LC) \in \Re_+^{n\times n}. \tag{14}$$

If the matrix G is a Schur matrix then the error e_i (12) will approach zero and $\hat{x}_i$ will approach x_i, i.e.:

$$\forall \hat{x}_0 \in \Re_+^n, \quad \lim_{i\to\infty}[x_i - \hat{x}_i] = 0, \tag{15}$$

and the observer (9) is asymptotically stable.

The problem of synthesis of the observer (9) for the positive system (1), (2) we can formulate as follows:

Given the matrix $A_{\overline{\alpha}}$ and C o (1), (2). We are looking for the gain matrix $L \in \Re_+^{n\times p}$ of the observer such that the matrix G (14) is a Schur matrix with nonnegative elements.

In the control theory of the standard continuous-time or discrete-time system a method which is frequently used to finding of the matrix L is Ackermann's formula [15, 16].

By generalization of conditions given in [10] for the positive discrete-time systems with integer order we can write the following theorem:

Theorem 2. The following statements are equivalent:

i) There exists a positive observer (9) of the positive fractional system (1), (2).

ii) There exists a matrix $L \in \Re_{+}^{n\times p}$ such that $LC > 0$, $G > 0$ and G (14) is a Schur matrix.

iii) The following LP problem is feasible:

$$\begin{cases} (A_{\alpha}^{T} - I_n)\lambda - C^T \sum_{i=1}^{n} z_1 < 0, \\ \lambda > 0, \\ c_i^T z_j \geq 0 \quad \text{for} \quad i,j = 1,\dots,n, \\ a_{ji}\lambda_j - c_i^T z_j \geq 0, \end{cases} \tag{16}$$

where $\lambda = [\lambda_1 \quad \dots \quad \lambda_n] \in \Re^n$, $z = [z_1 \dots z_n] \in \Re^n$.

Moreover, a matrix L satisfying the statement ii) can be calculated as:

$$L = \begin{bmatrix} \frac{z_1}{\lambda_1} & \dots & \frac{z_n}{\lambda_n} \end{bmatrix}^T. \tag{17}$$

where the variables λ_i, z_i can be any feasible solution to the above LP problem.

Proof. See [10], for example.

The above linear programming problem can be formulated and solved for example in the MATLAB package with Optimization toolbox.

Linear matrix inequality method

The linear matrix inequality (LMI) has the following canonical form [17]:

$$F(x) := F_0 + \sum_{i=1}^{m} x_i F_i \succ 0, \tag{18}$$

where $x \in \Re^m$ is the variable and the symmetric matrices $F_i = F_i^T \in \Re^{n\times n}$ $i = 0,1,\dots,m$ are given. Thus, $F(x)$ is an affine function of the elements of x.

The inequality means that $F(x)$ is a positive define matrix, that is:

$$z^T F(x) z \succ 0, \quad \forall z \neq 0, \quad z \in \Re^n. \tag{19}$$

The form (18) is a strict LMI and is feasible if the set $\{x \mid F(x) \succ 0\}$ is nonempty. Any feasible nonstrict LMI can be reduced to an equivalent strict LMI that is feasible by eliminating implicit equality constraints and then reducing the resulting LMI by removing any constant nullspace.

It is well-kown [2] that the positive discrete-time (integer order) system is asymptotically stable ($A \in \Re_{+}^{n\times n}$ is a Schur matrix) if and only if the following inequalities with respect to the diagonal matrix variable P are satisfied:

$$P - A^T P A \succ 0, \quad P = \operatorname{diag}(p_1,\dots,p_n) \succ 0. \tag{20}$$

The problem of the observer synthesis for the system (1), (2) we can reduce to finding the matrix $L \in \Re_{+}^{n\times p}$, such that the inequality

$$P - G^T P G \succ 0, \quad P = \operatorname{diag}(p_1,\dots,p_n) \succ 0 \tag{21}$$

is satisfied, where G has the form (14).

Using Schur complement formula and applying the congruence transformation into (21) we obtain:

$$\begin{bmatrix} P & G^T P \\ P G^T & P \end{bmatrix} \succ 0. \tag{22}$$

Premultiplying both sides (22) by $P^{-1} \succ 0$ and taking $Q = P^{-1}$ we get the final LMI condition in the form:

$$\begin{matrix} \begin{bmatrix} Q & QG - YC \\ G^T Q & Q \end{bmatrix} \succ 0 \\ (QG - YC) \geq 0 \end{matrix} \tag{23}$$

where $Y = P^{-1} L$.

We can sum up the above considerations in the following theorem based on the results given in [9]:

Theorem 3. There exists an asymptotically stable positive observer (9) of the system (1), (2) if and only if the condition (23) is satisfied with respect to the matrix variables $Q = P^{-1}$ $(P = \operatorname{diag}(p_1,\dots,p_n))$ and $Y \in \Re_{+}^{n\times p}$.
The gain matrix $L \in \Re_{+}^{n\times p}$ of the observer can be computed from:

$$L = Y Q^{-1}. \tag{24}$$

Proof. See [9], for example.

The linear matrix inequality (23) can be formulated and solved in the MATLAB package together with public domain software: SeDuMi solver and YALMIP parser.

3. Example

Let us consider the fractional system defined by equations (1) and (2) with matrices:

$$A = \begin{bmatrix} -0.05 & 0.75 \\ 0 & -0.35 \end{bmatrix}, B = \begin{bmatrix} 0.52 \\ 0.15 \end{bmatrix}, C = [0.45 \quad 1.05,] \tag{25}$$

and $\alpha_1 = 0.2$, $\alpha_2 = 0.4$.

Design a full-order positive observer for the above system.

We will check the positivity of the system with (25). Using (6) we obtain:

$$A_{\overline{\alpha}} = A + \overline{\alpha}_1 = A + diag\left[\begin{pmatrix}\alpha_1\\1\end{pmatrix} \quad \begin{pmatrix}\alpha_2\\1\end{pmatrix}\right] = \\ = \begin{bmatrix}0.15 & 0.75\\0 & 0.05\end{bmatrix} \in \Re_+^{2\times 2}. \tag{26}$$

Thus, by Theorem 1 the considered fractional system with $\alpha_1 = 0.2$, $\alpha_2 = 0.4$. is positive. Two methods are used to design the observer: LP method and LMI method.

– LP method

According with (11) the vector x has the form:

$$x = \begin{bmatrix}\lambda_1 & \lambda_2 & z_1 & z_2\end{bmatrix}^T, \tag{27}$$

The conditions we can write in the following forms:

$$\left(\begin{bmatrix}0.15 & 0.75\\0 & 0.05\end{bmatrix}^T - \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\right)\begin{bmatrix}\lambda_1\\\lambda_2\end{bmatrix} - \begin{bmatrix}0.45\\1.05\end{bmatrix}\begin{bmatrix}z_1 & z_2\end{bmatrix} < 0 \tag{28}$$

$$\begin{bmatrix}-1 & 0\\0 & -1\end{bmatrix}\begin{bmatrix}\lambda_1\\\lambda_2\end{bmatrix} < 0 \tag{29}$$

$$\begin{bmatrix}-0.45 & -0.45\\-1.05 & -1.05\\0 & 0\\0 & 0\end{bmatrix}\begin{bmatrix}z_1\\z_2\end{bmatrix} \le 0 \tag{30}$$

$$\begin{bmatrix}-0.15 & 0\\0 & 0\\-0.75 & 0\\0 & -0.05\end{bmatrix}\begin{bmatrix}z_1\\z_2\end{bmatrix} + \begin{bmatrix}0.45 & 0.45\\1.05 & 1.05\\0 & 0\\0 & 0\end{bmatrix}\begin{bmatrix}z_1\\z_2\end{bmatrix} \le 0 \tag{31}$$

Solving (11) with (28) – (31) and with the zero vector b of appropriate dimensions in the MATLAB environment and using *m*-function *linprog* we obtain the gain matrix (17) in the form:

$$L_{(LP)} = \begin{bmatrix}0.33\\0\end{bmatrix} \in \Re_+^2. \tag{32}$$

It is easy to check that the matrix (14) with (32) is a Schur matrix which has the structure:

$$G_{(LP)} = A_{\overline{\alpha}} - L_{(LP)}C = \begin{bmatrix}0 & 0.4\\0 & 0.05\end{bmatrix} \in \Re_+^{2\times 2}. \tag{33}$$

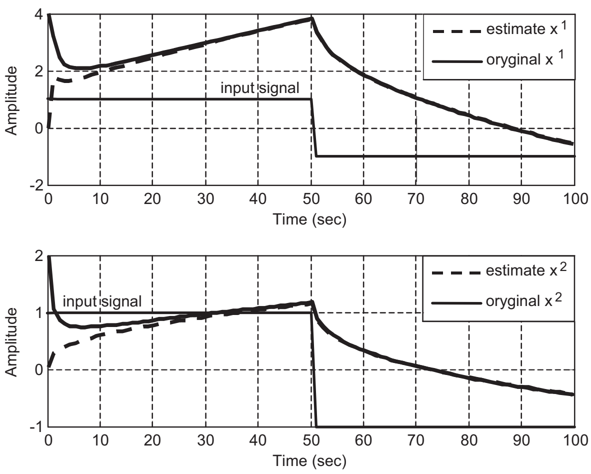

Fig. 1. State variables (solid line) and their estimates (dashed line)

Rys. 1. Zmienne stanu (linia ciągła) oraz ich estymaty (linia przerywana)

Thus, the observer is positive and asymptotically stable. The results of estimation of the state variables of the considered system (25) with the sampling period of $0.1\,\text{sec}$ are shown in fig. 1.

The initial conditions of the system and the observer have the form:

$$x_0 = \begin{bmatrix}x_0^1\\x_0^2\end{bmatrix}^T = \begin{bmatrix}4\\2\end{bmatrix}, \quad \hat{x}_0 = \begin{bmatrix}\hat{x}_0^1\\\hat{x}_0^2\end{bmatrix}^T = \begin{bmatrix}0\\0\end{bmatrix}. \tag{34}$$

It is easy to check that the error e_i (12) is equal to zero.

– LMI method

Using MATLAB package together with SeDuMi solver and YALMIP parser it is easy to check that for the matrix:

$$Q = P^{-1} = \begin{bmatrix}0.83 & 0\\0 & 1.25\end{bmatrix}, \quad Y = \begin{bmatrix}0.12\\0\end{bmatrix}, \tag{35}$$

the inequality (23) is satisfied. Computing the gain matrix of the observer from (24) we get:

$$L_{(LMI)} = YQ^{-1} = \begin{bmatrix}0.15\\0\end{bmatrix} \in \Re_+^2. \tag{36}$$

With (36) the matrix G (14) is a Schur matrix and has the following form:

$$G_{(LMI)} = A_{\overline{\alpha}} - L_{(LMI)}C = \begin{bmatrix}0.08 & 0.59\\0 & 0.05\end{bmatrix} \in \Re_+^{2\times 2}. \tag{37}$$

The results of estimation of the state variables of the system (25) with the sampling period of $0.1\,\text{sec}$ are shown in fig. 2.

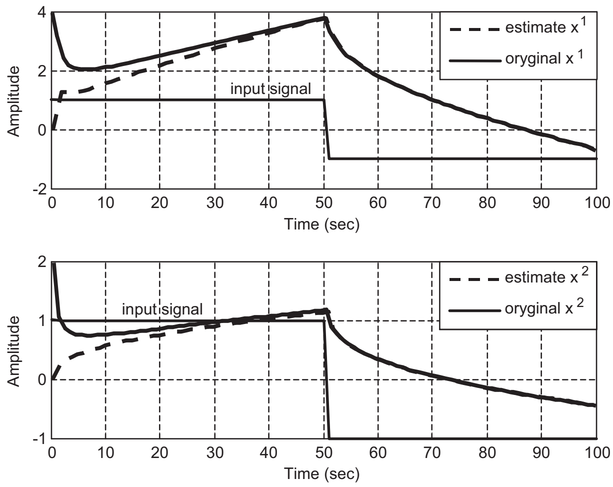

Fig. 2. State variables (solid line) and their estimates (dashed line)

Rys. 2. Zmienne stanu (linia ciągła) oraz ich estymaty (linia przerywana)

From the obtained results it follows that the state variables of the positive fractional system (25) are estimated correctly. Moreover, from Fig.1 it follows that in the considered case the dynamics of observer projected by the use of LP method is faster than the observer projected by the use of LMI method. It follows from the fact that the spectrum of (33) is $\sigma(G_{(LP)}) = \{0, 0.05\}$ and the spectrum of (37) is $\sigma(G_{(LMI)}) = \{0.08, 0.05\}$ It is well-known [10, 19] that if the eigenvalues of the matrix of asymptotic stable dynamical system are located the nearer of the coordinate origin (in the z plane) then the transient processes (state variables) tends faster to zero.

4. Concluding remarks

In the paper the problem of observer synthesis for positive linear discrete-time systems with different fractional order have been considered. It has been shown that proposed conditions of the existence of asymptotic stable positive observer are solvable in term of linear programming and linear matrix inequality problems. An example to illustrate the effectiveness and correctness of the obtained results has been given.

The presented considerations can be easily extended for reduced-order observer synthesis for standard and positive discrete-time fractional systems with different fractional orders.

Acknowledgment

This work was supported by the National Science Center in Poland under grant NN 514 6389 40.

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Synteza obserwatora układów dyskretnych o różnych niecałkowitych rzędach

Streszczenie: W pracy rozpatrzono problem syntezy obserwatorów dla dodatnich układów dyskretnych różnych niecałkowitych rzędów w równaniu stanu. Wykorzystując podejście oparte na typowym zadaniu programowania liniowego (LP) oraz zadaniu sformułowanym w ramach liniowych nierówności macierzowych (LMI) pokazano, że jest możliwe uzyskanie dodatniego asymptotycznie stabilnego obserwatora. Są to warunki dostateczne, alternatywne w stosunku do podanych w [5,18] dla układów niedodatnich. Zaprojektowany obserwator poprawnie estymuje (odtwarza) zmienne stanu przyjętego do rozważań dyskretnego układu niecałkowitego rzędu. Wyniki obliczeniowe uzyskano w środowisku programowym MATLAB z wykorzystaniem biblioteki Optimization oraz pakietów SeDuMi [20] i YALMIP [14]. Rezultaty symulacyjne uzyskano przy wykorzystaniu dodatkowej biblioteki Fractional States Space Toolkit [18].

Słowa kluczowe: rząd niecałkowity, układ, dodatni, dyskretny, obserwator, programowanie liniowe, liniowa nierówność macierzowa

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