Stability of fractional discrete-time linear scalar systems with one delay

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Abstract: In the paper the problems of practical stability and asymptotic stability of fractional discrete-time linear scalar systems with one constant delay are addressed. Standard and positive systems are considered. New conditions for practical stability and for asymptotic stability are established.

Keywords: linear system, discrete-time, scalar, fractional, delay, stability.

1. Introduction

The problems of analysis and synthesis of dynamic systems described by fractional order differential (or difference) equations have recently considerable attention, see monographs [2, 15, 16, 18, 19, 21, 23] and papers [9, 11, 17, 24], for example, and references therein.

The problem of stability of linear continuous-time fractional order systems has been considered in many paper, see for example [4, 5], Chapter 9 in [16], [20, 22], and references therein.

Stability problem of fractional order discrete-time linear systems is more complicated because asymptotic stability of such systems is equivalent to asymptotic stability of the corresponding infinite-dimensional discrete-time systems of natural order with delays [10]. In practical problems only bounded number of delays (called the length of practical implementation) can be considered. In this case the corresponding discrete-time linear system of natural order has finite number of delays and it is called the practical realization of fractional order system. Asymptotic stability of this system is called the practical stability of the fractional system. The conditions for practical stability with given length of practical implementation for standard fractional discrete-time systems were derived in [10, 13].

Simple necessary and sufficient conditions for practical stability and for asymptotic stability of positive discrete-time linear systems of fractional order were established in [3, 8, 14, 15, 16].

Recently, simple analytic conditions for practical stability and for asymptotic stability of a class of standard fractional order discrete-time linear systems were given in [6, 7].

The aim of this paper is to give the conditions for practical stability and for asymptotic stability of fractional discrete-time linear scalar systems with one constant delay, standard and positive. To the best knowledge of the author, such conditions have not been established yet.

The following notations will be used: $\Re$ – the set of real numbers; Z_+ – the set of non-negative integers.

2. Problem formulation

Let us consider the fractional order scalar discrete-time linear system with delay described by the homogeneous state equation (for $i \in Z_+$)

$$\Delta^{\alpha} x_{i+1} = a_0 x_i + a_1 x_{i-1}, \quad 0 < \alpha < 1, \tag{1}$$

with the initial conditions x_{-l} $(l = 0,1)$, where $x_i \in \Re$ is the state variable, a_0 and a_1 are constant coefficients and

$$\Delta^{\alpha} x_i = \sum_{k=0}^{i} (-1)^k \binom{\alpha}{k} x_{i-k} \tag{2}$$

with

$$\binom{\alpha}{k} = \begin{cases} 1 & \text{for } k = 0 \\ \dfrac{\alpha(\alpha-1)\cdots(\alpha-k+1)}{k!} & \text{for } k > 0 \end{cases} \tag{3}$$

is the fractional difference of order $\alpha \in (0,1)$ of the discrete-time function x_i [14–16].

From (2) and (3) it follows that $\Delta^0 x_i = x_i$ and $\Delta^1 x_i = x_i - x_{i-1}$. This means that the equation (1) for $\alpha = 0$ and $\alpha = 1$ takes the following forms:

– for $\alpha = 0$

$$x_{i+1} = a_0 x_i + a_1 x_{i-1}, \tag{4}$$

– for $\alpha = 1$

$$x_{i+1} = (1 + a_0) x_i + a_1 x_{i-1}. \tag{5}$$

From the above and classical stability theory of discrete-time linear systems we have the following lemmas.

Lemma 1. The fractional system (1) with $\alpha = 0$ (the discrete-time system (4), equivalently) is asymptotically stable if and only if

$$a_1 < 1 - a_0, \quad a_1 < 1 + a_0, \quad a_1 > -1. \tag{6}$$

Lemma 2. The fractional system (1) with $\alpha = 1$ (the discrete-time system (5), equivalently) is asymptotically stable if and only if

$$a_1 < -a_0, \quad a_1 < 2 + a_0, \quad a_1 > -1. \tag{7}$$

Using (2) we may write the equation (1) in the form

$$x_{i+1} = (a_0 + \alpha)x_i + \sum_{k=1}^{i} c_k(\alpha)x_{i-k} + a_1 x_{i-1}, \tag{8}$$

where

$$c_k(\alpha) = (-1)^k \binom{\alpha}{k+1}, \quad k = 1,2,\ldots . \tag{9}$$

The equation (8) describes the discrete-time linear system with increasing number of delays.

The fractional discrete-time system (1) is called positive (internally), if $x_i \geq 0$ ($i \in Z_+$) for any initial conditions $x_{-l} \geq 0$ $(l = 0,1)$.

The fractional system (1) (or (8), equivalently) is positive if and only if [14–16]

$$a_0 + \alpha \geq 0 \ \text{ and } \ a_1 + c_1(\alpha) \geq 0. \tag{10}$$

The coefficients (9) can be computed by the following simple algorithm suitable for computer programming [8]

$$c_{k+1}(\alpha) = c_k(\alpha)\frac{k+1-\alpha}{k+2}, \qquad k = 1, 2,\ldots \tag{11}$$

where $c_1(\alpha) = 0.5\alpha(1-\alpha)$.

From (11) it follows that $c_k(\alpha) > 0$ for $\alpha \in (0,1)$ and $k = 1, 2,\ldots$ Moreover, the coefficients $c_k(\alpha)$ strongly decrease for increasing k. Therefore, in (8) we can assume that k is bounded by some natural number L. This number is called the length of practical implementation or the length of finite memory. In this case the equation (8) takes the form (for $i>L$)

$$x_{i+1} = (a_0 + \alpha)x_i + a_1 x_{i-1} + \sum_{k=1}^{L} c_k(\alpha)x_{i-k}. \tag{12}$$

The equation (12) describes a linear discrete-time system with L delays in state.

The time-delay system (12) is called the practical realization of the fractional system (1).

Definition 1 [14]. The fractional system (1) is called practically stable if the system (12) is asymptotically stable.

Definition 2 [14]. The fractional system (1) is called asymptotically stable if the system (12) is practically stable for $L \to \infty$.

From Definition 1 and theory of asymptotic stability of discrete-time linear systems we have the following theorem.

Theorem 1. The fractional system (1) with length L of practical implementation is practically stable if and only if

$$w(z) \neq 0, \qquad |z| \geq 1, \tag{13}$$

where

$$w(z) = z - (a_0 + \alpha) - a_1 z^{-1} - \sum_{k=1}^{L} c_k(\alpha) z^{-k}. \tag{14}$$

The characteristic equation $w(z) = 0$ of the system (12) can be written in the form (for $z \neq 0$)

$$z^{L+1} - (a_0 + \alpha)z^L - a_1 z^{L-1} - \sum_{k=1}^{L} c_k(\alpha) z^{L-k} = 0. \tag{15}$$

From the above it follows that to checking the practical stability of the fractional system (1) we can apply the classical methods for asymptotic stability analysis of the systems (12) with delays. However, these methods may be inconvenient with respect to high degree of the equation (15) for a large length L of practical implementation.

The aim of the paper is to give the simple conditions for practical stability and for asymptotic stability of the fractional system (1), standard and positive.

3. Solution of the problem

We apply the D-decomposition method of Nejmark [12, 22] for stability investigation of the system (1) in dependence of values the coefficients a_0 and a_1. According to this method, the plane (a_0, a_1) is divided by the boundaries of D-decomposition into such regions $D(p)$, that any point in $D(p)$ corresponds to such values of a_0 and a_1 that polynomial (14) has exactly p roots z_r satisfying the condition $|z_r| > 1$. The region $D(0)$, if it exists, is the stability region of the polynomial (14).

Any point on the boundaries of D-decomposition corresponds to polynomial (14) (or equation (15)) with at least one root on the stability boundary, i.e. $z_r = -1$ or $z_r = 1$ (the real roots boundary) or $z_r = \exp(j\omega_r)$, $j^2 = -1$ (the complex roots boundary).

Solving with respect to a_1 the equations $w(1) = 0$ and $w(-1) = 0$, where $w(z)$ have the form (14) one obtains

$$a_1 = 1 - a_0 - \alpha - \sum_{k=1}^{L} c_k(\alpha), \tag{16}$$

$$a_1 = 1 + a_0 + \alpha + \sum_{k=1}^{L} c_k(\alpha)(-1)^{-k}. \tag{17}$$

Lemma 3. If $L \to \infty$ then the formulae (16) and (17) take the following forms

$$a_1 = -a_0, \tag{18}$$

$$a_1 = 2^{\alpha} + a_0. \tag{19}$$

Proof. In [10, 15, 16] and [7], respectively, it has been proved that

$$\sum_{k=1}^{\infty} c_k(\alpha) = 1 - \alpha \tag{20}$$

and

$$1 + \alpha + \sum_{k=1}^{\infty} (-1^k) c_k(\alpha) = 2^{\alpha}. \tag{21}$$

Substitution (20) and (21) into (16) and (17) gives (18) and (19). This completes the proof.

In the plane (a_0, a_1) the straight lines (16), (17) (for $L < \infty$) and (18), (19) (for $L = \infty$) are the real roots boundaries.

Solving with respect to a_0 and a_1 the equation

$$w(e^{j\omega}) = e^{j\omega} - (a_0 + \alpha) - a_1 e^{-j\omega} - \sum_{k=1}^{L} c_k(\alpha) e^{-jk\omega} = 0, \qquad (22)$$

we obtain

$$a_0(\omega) = -\alpha + 2\cos(\omega) - \sum_{k=1}^{L} c_k(\alpha)\cos(\omega k) - \operatorname{ctg}(\omega) \sum_{k=1}^{L} c_k(\alpha)\sin(\omega k), \qquad (23)$$

$$a_1(\omega) = -1 - \frac{\sum_{k=1}^{L} c_k(\alpha)\sin(\omega k)}{\sin(\omega)}. \qquad (24)$$

The curve with parametric description (23), (24) for $\omega \in [0, 2\pi]$ is the complex roots boundary in the plane (a_0, a_1) of the polynomial (14).

From (23) and (24) for $\omega = 0$ and $\omega = \pi$ we have (applying the L'Hospital rule)

– for $\omega = 0$

$$a_0(0) = 2 - \alpha + \sum_{k=1}^{L} c_k(\alpha)(k-1),$$

$$a_1(0) = -1 - \sum_{k=1}^{L} k c_k(\alpha), \qquad (25)$$

– for $\omega = \pi$

$$a_0(\pi) = -2 - \alpha + \sum_{k=1}^{L} (k-1) c_k(\alpha)\cos(k\pi),$$

$$a_1(\pi) = -1 + \sum_{k=1}^{L} k c_k(\alpha)\cos(k\pi). \qquad (26)$$

From Lemma 1 we have that in the plane (a_0, a_1) boundaries of the asymptotic stability region of the system (1) with $\alpha = 0$ are as follows: $a_1 = 1 - a_0$, $a_1 = 1 + a_0$ and $a_1 = -1$. This region is the triangle Δ_0 with the vertices $(-2,-1)$, $(2,-1)$ and $(0,1)$. The triangle Δ_0 is shown in figure 1 (boundary 1).

Similarly, from Lemma 2 we obtain that in the plane (a_0, a_1) the asymptotic stability region of the system (1) for $\alpha = 1$ is the triangle Δ_1 with the vertices $(-3,-1)$, $(1,-1)$ and $(-1,1)$.

Denote by $S(\alpha, L)$ the open region in the (a_0, a_1) plane with boundaries determined by segments of the straight lines (16), (17) and by segment of the curve with parametric description (23), (24) for $\omega \in \Omega \subseteq [0, 2\pi]$.

The regions $S(\alpha, L)$ for $L = 10$, $\alpha = 0.5$ and $L = 10$, $\alpha = 0.9$ are shown in figure 1. The complex roots boundary (23), (24) is plotted for $\omega \in \Omega = [0.14\pi, \pi]$ for $\alpha = 0.5$ and $\omega \in \Omega = [0, \pi]$ for $\alpha = 0.9$.

Figure 2 shows the regions $S(\alpha, L)$ for $\alpha = 0.5$, $L = 10$ and $\alpha = 0.5$, $L = 10\,000$, where segments of the curve (23), (24) are plotted for $\omega \in \Omega = [0.14\pi, \pi]$ for $L = 10$ and $\omega \in \Omega = [0.195\pi, \pi]$ for $L = 10\,000$.

Theorem 2. The fractional system (1) with given length L of practical implementation is practically stable if and only if values of the coefficients a_0 and a_1 correspond to the points lying in the practical stability region $S(\alpha, L)$.

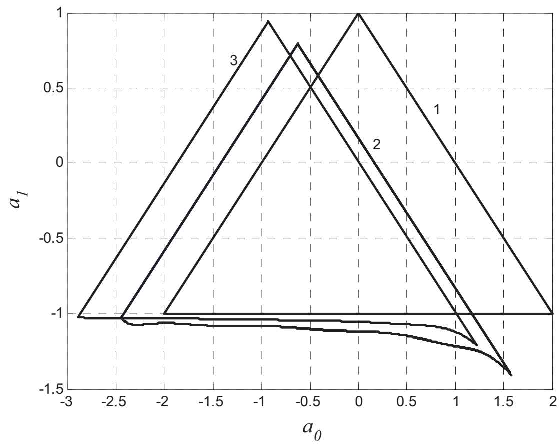

Fig. 1. The asymptotic stability region of the system (1) with $\alpha = 0$ (boundary 1) and regions $S(\alpha, L)$ for: $\alpha = 0.5$; $L = 10$ (boundary 2) and $\alpha = 0.9$; $L = 10$ (boundary 3)

Rys. 1. Obszar stabilności asymptotycznej układu (1) dla $\alpha = 0$ (brzeg 1) oraz obszary $S(\alpha, L)$ dla $\alpha = 0{,}5$; $L = 10$ (brzeg 2) oraz $\alpha = 0{,}9$; $L = 10$ (brzeg 3)

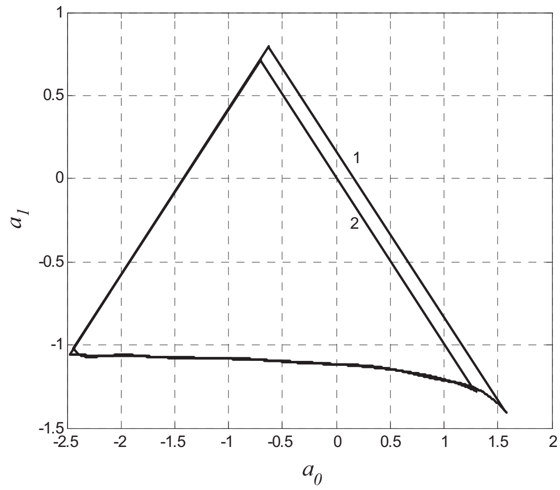

Fig. 2. The regions $S(\alpha, L)$ for $\alpha = 0.5$ and $L = 10$ (boundary 1) and $\alpha = 0.5$, $L = 10\,000$ (boundary 2)

Rys. 2. Obszary $S(\alpha, L)$ dla $\alpha = 0{,}5$, $L = 10$ (brzeg 1) oraz $\alpha = 0{,}5$, $L = 10\,000$ (brzeg 2)

Proof. We show that the region $S(\alpha, L)$ is the asymptotic stability region of the system (12) and, by Definition 1, is also the region of practical stability of the fractional system (1) with fixed $\alpha \in (0,1)$ and $L \ge 2$.

According to the D-decomposition method, it is sufficient to prove that in $S(\alpha, L)$ exists at least one pair of values of a_0 and a_1 for which all the roots z_r of the equation (15) satisfy the condition $|z_r| < 1$, $r = 1, 2, \ldots, L+1$.

From Figure 1 (see also figures 2 and 3) and Lemmas 1 and 2 it follows that the point with $a_0 = a_1 = 0$ lies in the open region $S(\alpha, L)$ for all fixed $\alpha \in (0,1)$ and for all $L \ge 2$.

The equation (15) for $a_0 = a_1 = 0$ has the form

$$z^{L+1} - \alpha z^L - c_1(\alpha) z^{L-1} - \sum_{k=2}^{L} c_k(\alpha) z^{L-k} = 0. \qquad (27)$$

In [1] it has been shown that all roots of the polynomial $z^n + a_{n-1}z^{n-1} + \cdots + a_1 z + a_0$ have absolute values less than 1 if $1 > |a_{n-1}| + \cdots + |a_1| + |a_0|$. This condition for the equation (27) has the form

$$1 > \alpha + \sum_{k=1}^{L} c_k(\alpha). \tag{28}$$

Using the formula (20) from (28) one obtains

$$1 - \alpha - \sum_{k=1}^{L} c_k(\alpha) > 1 - \alpha - \sum_{k=1}^{\infty} c_k(\alpha) = 0. \tag{29}$$

Hence, the condition (28) holds and the characteristic equation (15) for $a_0 = a_1 = 0$ and any $\alpha \in (0,1)$ has $L+1$ roots which satisfy the condition $|z_r| < 1$ $(r = 1,2,\ldots,L+1)$. This means, according to the D-decomposition method, that open region $S(\alpha, L)$ is the asymptotic stability region of the system (12) and also is the practical stability region of the fractional system (1) with $\alpha \in (0,1)$. This completes the proof.

From figures 1 and 2 it follows that for any fixed $\alpha \in (0,1)$ and $L \geq 2$ in the region $S(\alpha, L)$ exists a triangle T_L which sides are segments of the straight lines (16), (17) and a segment of the straight line $a_1 = -1$.

All points lying in the open triangle T_L satisfy the following inequalities:

$$a_1 - 1 + a_0 + \alpha + \sum_{k=1}^{L} c_k(\alpha) < 0, \tag{30}$$

$$a_1 - 1 - a_0 - \alpha - \sum_{k=1}^{L} c_k(\alpha)(-1)^{-k} < 0, \tag{31}$$

$$a_1 > -1. \tag{32}$$

It is easy to check that triangle T_L has the vertices $V_p = (a_{0p}, a_{1p})$, $p = 1,2,3,$ where

$$a_{01} = -\alpha - \sum_{k=2,4,6,\ldots}^{L} c_k(\alpha), \quad a_{11} = 1 - \sum_{k=1,3,5,\ldots}^{L} c_k(\alpha), \tag{33}$$

$$a_{02} = -2 - \alpha - \sum_{k=1}^{L} c_k(\alpha)(-1)^k, \; a_{12} = -1, \tag{34}$$

$$a_{03} = 2 - \alpha - \sum_{k=1}^{L} c_k(\alpha), \quad a_{13} = -1. \tag{35}$$

From the above considerations and Theorem 2 we have the following lemma.

Lemma 4. The fractional system (1) with given length L of practical implementation is practically stable if values of the coefficients a_0 and a_1 correspond to the points lying in the triangle T_L, i.e. the coefficients a_0 and a_1 satisfy the inequalities (30)–(32).

If $L \to \infty$ then from Definition 2 and Lemmas 3 and 4 one obtains the following lemma.

Lemma 5. The fractional system (1) is asymptotically stable if the coefficients a_0 and a_1 satisfy the inequalities

$$a_1 + a_0 < 0, \quad a_1 - a_0 - 2^{\alpha} < 0, \quad a_1 > -1. \tag{36}$$

The coefficients a_0 and a_1 satisfy the inequalities (36) if and only if all points (a_0, a_1) lie in the triangle T_∞ with the vertices $V_p^\infty = (a_{0p}^\infty, a_{1p}^\infty)$, $p = 1,2,3,$ and sides (straight lines segments) $[V_1^\infty V_2^\infty]$, $[V_2^\infty V_3^\infty]$ and $[V_1^\infty V_3^\infty]$, where

$$a_{01}^\infty = -2^{\alpha-1}, \quad a_{11}^\infty = 2^{\alpha-1}, \tag{37}$$

$$a_{02}^\infty = -1 - 2^{\alpha}, \quad a_{12}^\infty = -1, \tag{38}$$

$$a_{03}^\infty = 1, \quad a_{13}^\infty = -1. \tag{39}$$

The triangle T_L with $\alpha = 0.5$, $L = 10$ and triangle T_∞ with $\alpha = 0.5$ are shown in figure 3.

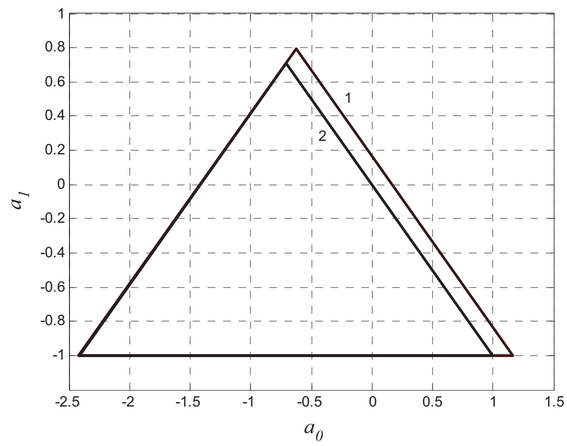

Fig. 3. Triangle T_L with $\alpha = 0.5$, $L = 10$ (boundary 1) and triangle T_∞ with $\alpha = 0.5$ (boundary 2)

Rys. 3. Trójkąt T_L dla $\alpha = 0{,}5$, $L = 10$ (brzeg 1) oraz trójkąt T_∞ dla $\alpha = 0{,}5$ (brzeg 2)

Now we consider the stability problem of the positive system (1) (the condition (10) holds).

Theorem 3. The positive fractional system (1) with given length L of practical implementation is practically stable if and only if

$$1 - a_0 - a_1 - \alpha - \sum_{k=1}^{L} c_k(\alpha) > 0. \tag{40}$$

Proof. Practical stability of the positive fractional system (1) is equivalent to asymptotic stability of the positive system (12) with delays.

From [8] it follows that the positive system (12) is asymptotically stable if and only if the positive scalar system

$$x_{i+1} = a x_i, \quad i \in Z_+, \tag{41}$$

is asymptotically stable, where

$$a = a_0 + \alpha + a_1 + \sum_{k=1}^{L} c_k(\alpha). \tag{42}$$

From the stability theory of positive systems ([15, 16], see also [6]) it follows that the positive scalar system (41) is asymptotically stable if and only if $1 - a > 0$. This condition for coefficient a defined by (42) has the form (40) and the proof is complete.

Theorem 4. The positive fractional system (1) is asymptotically stable if and only if

$$a_0 + a_1 < 0. \quad (43)$$

Proof. The proof follows from Theorem 3 for $L \to \infty$ and equality (20).

Note that asymptotic stability of the positive fractional system (1) does not depend on the fractional order $\alpha \in (0,1)$. On the fractional order α depends only the positivity condition (10).

The region of practical stability with $L = 10$ (triangle A_1BC_1) and the region of asymptotic stability (triangle ABC) of the positive fractional system (1) with $\alpha = 0.5$ are shown in figure 4.

For any fractional order $\alpha \in (0,1)$ coordinates of vertices of these triangles are as follows:

A: $a_0 = -\alpha, \quad a_1 = \alpha,$

B: $a_0 = -\alpha, \quad a_1 = -c_1(\alpha),$

C: $a_0 = c_1(\alpha), \quad a_1 = -c_1(\alpha),$

A_1: $a_0 = -\alpha, \quad a_1 = 1 + \sum_{k=1}^{L} c_k(\alpha),$

C_1: $a_0 = 1 + c_1(\alpha) - \alpha + \sum_{k=1}^{L} c_k(\alpha), \quad a_1 = -c_1(\alpha).$

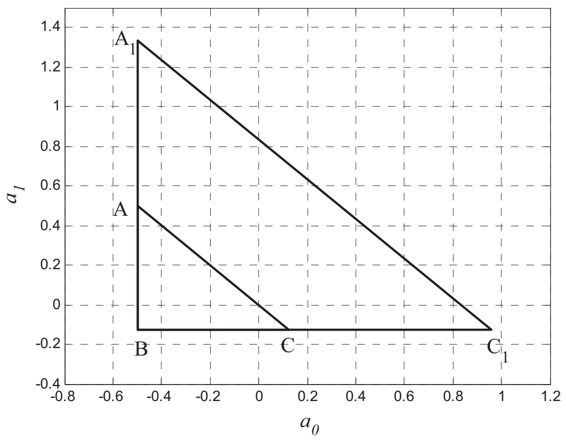

Fig. 4. Stability regions of the positive system (1) with $\alpha = 0.5$: region of asymptotic stability (triangle ABC); region of practical stability with $L = 10$ (triangle A_1BC_1)

Rys. 4. Obszary stabilności dodatniego układu (1) dla $\alpha = 0{,}5$: obszar stabilności asymptotycznej (trójkąt ABC); obszar stabilności praktycznej dla $L = 10$ (trójkąt A_1BC_1)

4. Concluding remarks

The problems of practical stability and asymptotic stability of the discrete-time linear system (1) of fractional order $0 < \alpha < 1$, standard and positive, have been addressed.

In the case of standard systems, necessary and sufficient condition for practical stability has been established in Theorem 2. Based on this condition, simple analytic sufficient conditions for practical stability and for asymptotic stability have been given in Lemmas 4 and 5.

In the case of positive systems, simple analytic necessary and sufficient analytic conditions for practical stability and for asymptotic stability have been given in Theorems 3 and 4.

The proposed conditions are original and have not been established yet.

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Stabilność dyskretnych skalarnych układów liniowych niecałkowitego rzędu z jednym opóźnieniem

Streszczenie: Rozpatrzono problem stabilności liniowych skalarnych układów dyskretnych niecałkowitego rzędu z jednym opóźnieniem zmiennych stanu. Wykorzystując metodę podziału D, podano graficzne warunki konieczne i wystarczające praktycznej stabilności. Bazując na tych warunkach, sformułowano proste analityczne warunki wystarczające stabilności praktycznej oraz stabilności asymptotycznej. W przypadku układów dodatnich podano proste analityczne warunki konieczne i wystarczające stabilności praktycznej oraz stabilności asymptotycznej.

Słowa kluczowe: układ liniowy, dyskretny, skalarny, niecałkowitego rzędu, opóźnienie, stabilność.

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