Positive realization of fractional discrete-time linear systems with delays

Łukasz Sajewski

Faculty of Electrical Engineering, Białystok University of Technology

Abstract: The positive realization problem for single-input single-output fractional discrete-time linear systems with delays in state vector and input is formulated and a method for finding a positive realization of a given proper transfer function is proposed. Sufficient conditions for the existence of a positive realization of this class of linear systems are established. A procedure for computation of a positive realization is proposed and illustrated by a numerical example.

Keywords: fractional, positive, delay, realization, existence, computation

1. Introduction

In positive systems inputs, state variables and outputs take only non-negative values. Examples of positive systems are industrial processes involving chemical reactors, heat exchangers and distillation columns, storage systems, compartmental systems, water and atmospheric pollution models. A variety of models having positive linear systems behavior can be found in engineering, management science, economics, social sciences, biology and medicine, etc.

Positive linear systems are defined on cones and not on linear spaces. Therefore, the theory of positive systems is more complicated and less advanced. An overview of state of art in positive systems theory is given in the monographs [3, 12]. The realization problem for positive discrete-time and continuous-time systems without and with delays was considered in [1, 3, 5, 6, 9, 10, 13].

The first definition of the fractional derivative was introduced by Liouville and Riemann at the end of the 19th century [17, 18]. This idea has been used by engineers for modeling different process [2, 4, 15, 20, 21]. Mathematical fundamentals of fractional calculus are given in the monographs [16-19, 22]. The fractional order controllers have been developed in [20, 23]. A generalization of the Kalman filter for fractional order systems has been proposed in [25]. A class of positive fractional discrete-time linear system with and without delays has been introduced in [7, 14]. The realization problem for positive fractional systems was considered in [8, 11, 14, 24].

The main purpose of this paper is to present a method for computation of a positive realization of SISO fractional discrete-time linear systems with delays in state vector and input for given proper transfer function. Sufficient conditions for the existence of a positive realization of this class of systems will be established and a procedure for computation of a positive realization will be proposed.

The paper is organized as follows. In section 2 basic definition and theorem concerning positive fractional systems with delays are recalled. Also in this section using the zet transform the transfer matrix (function) of the fractional linear systems is derived and the positive realization problem is formulated. Main result is given in section 3 where solution to the realization problem for given transfer function of the fractional discrete-time linear systems with delays is given. In the same section the sufficient conditions for the positive realization are derived and the procedure for computation of the positive realization is proposed. Concluding remarks are given in section 4.

The following notation will be used: $\Re$ the set of real numbers, $\Re^{n\times m}$ the set of $n\times m$ real matrices, $\Re_+^{n\times m}$ the set of $n\times m$ matrices with nonnegative entries and $\Re_+^n = \Re_+^{n\times 1}$, I_n the $n\times n$ identity matrix, $\mathcal{Z}[f(k)]$ · zet transform of the discrete-time function $f(k)$.

2. Preliminaries and problem formulation

Consider a fractional discrete-time linear system with q delays in state vector and input described by the equations

$$\Delta^\alpha x_{k+1} = \sum_{r=0}^{q}(A_r x_{k-r} + B_r u_{k-r}) \tag{2.1a}$$

$$y_k = Cx_k + Du_k\,,\quad k \in Z_+ \tag{2.1b}$$

where $x_k \in \Re^n$ is state vector $u_k \in \Re^m$ is input vector and $y_k \in \Re^p$ is output vector and $A_r \in \Re^{n\times n}$: r = 0,1,...q; $B \in \Re^{n\times m}$, $C \in \Re^{p\times n}$, $D \in \Re^{p\times m}$.

The fractional difference of $\alpha \in \Re$ order is defined by

$$\Delta^\alpha x_k = \sum_{j=0}^{k}(-1)^j \binom{\alpha}{j} x_{k-j} \tag{2.2a}$$

and

$$\binom{\alpha}{j} = \begin{cases} 1 \quad \text{for} \quad j=0 \\ \frac{\alpha(\alpha-1)\cdots}{j!} \quad \underline{-1)} \quad \text{for} \quad j=1,2,\ldots \end{cases} \tag{2.2b}$$

Using (2.2a) we can write the equation (2.1a) in the following form

$$x_{k+1} = \sum_{r=0}^{q}(A_r x_{k-r} + B_r u_{k-r}) + \sum_{j=1}^{k+1}(-1)^{j+1}\binom{\alpha}{j} x_{k-j+1} \tag{2.3}$$

Definition 2.1. The fractional system with delays in state vector and input (2.1) is called positive if and only if $x_k \in \Re_+^n$ and $y_k \in \Re_+^p$, $k \in Z_+$ for any initial conditions $x_h \in \Re_+^n$, $h = 0,-1,...,-q$ and all input sequences $u_k \in \Re_+^m$, $k \in Z_+$.

Theorem 2.1. [14] The fractional discrete-time linear system with q delays in state vector and input (2.1) is positive for $0 < \alpha < 1$ if and only if

$$A_{r\alpha} = A_r + c_{r+1} I_n \in \Re_+^{n\times n},\; c_r = (-1)^{r+1}\binom{\alpha}{r},\; B_r \in \Re_+^{n\times m},$$
$$r = 0,1,...,q;\; C \in \Re_+^{p\times n},\; D \in \Re_+^{p\times m}. \quad (2.4)$$

Proof is given in [14].

Substituting (2.2a) into (2.1a) we obtain

$$x_{k+1} = \sum_{r=0}^{q} A_{r\alpha} x_{k-r} + \sum_{j=q+1}^{k+1} c_j x_{k-j+1} + \sum_{r=0}^{q} B_r u_{k-r}$$
$$y_k = Cx_k + Du_k \quad k \in Z_+ \quad (2.5)$$

where

$$A_{r\alpha} = A_r + I_n(-1)^{r+2}\binom{\alpha}{r+1},\; c_j = (-1)^{j+1}\binom{\alpha}{j}. \quad (2.6)$$

Performing the zet transform on (2.5) we have

$$zX(z) - zx_0 = \sum_{r=0}^{q} A_{r\alpha} z^{-r}\left[X(z) + \sum_{l=-1}^{-r} x_l z^{-l}\right] + \sum_{j=q+1}^{k+1} c_j z^{1-j}\left[X(z) + \sum_{l=-1}^{1-j} x_l z^{-l}\right]$$
$$+ \sum_{r=0}^{q} B_r z^{-r}\left[U(z) + \sum_{l=-1}^{-r} u_l z^{-l}\right] \quad (2.7)$$
$$Y(z) = CX(z) + DU(z)$$

where $X(z) = \mathcal{Z}[x_k]$, $U(z) = \mathcal{Z}[u_k]$, $Y(z) = \mathcal{Z}[y_k]$. Now we can rewrite the equations (2.7) in the following form

$$\left[I_n z - \sum_{r=0}^{q} A_{r\alpha} z^{-r} - I_n \sum_{j=q+1}^{k+1} c_j z^{1-j}\right] X(z)$$
$$= zx_0 + \sum_{r=0}^{q} A_{r\alpha} z^{-r} \sum_{l=-1}^{-r} x_l z^{-l} + \sum_{j=q+1}^{k+1} c_j z^{1-j} \sum_{l=-1}^{1-j} x_l z^{-l} \quad (2.8)$$
$$+ \sum_{r=0}^{q} B_r z^{-r} U(z) + \sum_{r=0}^{q} B_r z^{-r} \sum_{l=-1}^{-r} u_l z^{-l}$$
$$Y(z) = CX(z) + DU(z)$$

and for zero initial conditions we have

$$X(z) = \left[I_n\left(z - \sum_{j=q+1}^{k+1} c_j z^{1-j}\right) - \sum_{r=0}^{q} A_{r\alpha} z^{-r}\right]^{-1} \sum_{r=0}^{-r} B_r z^{-r} U(z) \quad (2.9)$$
$$Y(z) = CX(z) + DU(z)$$

The transfer function of the system (2.1) is given by the equation

$$T(z) = \frac{Y(z)}{U(z)} = C\left[I_n(z - g_\alpha) - \sum_{r=0}^{q} A_{r\alpha} z^{-r}\right]^{-1} \sum_{r=0}^{q} B_r z^{-r} + D \quad (2.10a)$$
$$= T_{sp}(z) + D$$

where

$$A_{r\alpha} = A_r + I_n(-1)^{r+2}\binom{\alpha}{r+1},$$
$$g_\alpha = \sum_{j=q+1}^{k+1} c_j z^{1-j} = \sum_{j=q+1}^{k+1} (-1)^{j+1}\binom{\alpha}{j} z^{1-j}. \quad (2.10b)$$

and $T_{sp}(z)$ is the strictly proper transfer function.

In general case the transfer matrix is the function of the operators $w = z - g_\alpha$, z^{-r}, $r = 0,1,...,q$ and for single-input single-output (shortly SISO) systems the proper transfer function has the following form

$$T(w,z) = \frac{b_{n-1}(z)w^{n-1} + ... + b_1(z)w + b_0(z)}{w^n - a_{n-1}(z)w^{n-1} - ... - a_1(z)w - a_0(z)} + D \quad (2.11a)$$

$$b_k(z) = b_k^0 + b_k^1 z^{-1} + ... + b_k^q z^{-q},$$
$$a_k(z) = a_k^0 + a_k^1 z^{-1} + ... + a_k^q z^{-q}, \quad (2.11b)$$
$$k = 0,1,..., n-1$$

for known α.

Definition 2.2. The matrices (2.4) are called the positive realization of the transfer matrix $T(w,z)$ if they satisfy the equality (2.10a).

The realization problem can be stated as follows.

Given a proper rational transfer matrix $T(w,z) \in \Re^{p\times m}(w,z)$ and fractional order α, find its positive realization (2.4), where $\Re^{p\times m}(w,z)$ is the set of $p\times m$ rational matrices in w and z.

3. Problem solution for SISO systems

The essence of proposed method for solving of the realization problem for positive fractional discrete-time linear systems with delays will be presented on single-input single-output system.

Transfer function (2.10) can be written in the following form

$$T(w,z) = \frac{C(H_{ad}(w,z))[B_0 + B_1 z^{-1} + ... + B_q z^{-q}]}{\det H(w,z)} + D$$
$$= \frac{N(w,z)}{d(w,z)} + D \quad (3.1)$$

where

$$H(w,z) = [I_n w - A_{0\alpha} - A_{1\alpha} z^{-1} - ... - A_{q\alpha} z^{-q}]$$
$$N(w,z) = C(H_{ad}(w,z))[B_0 + B_1 z^{-1} + ... + B_q z^{-q}]$$
$$= n_{n-1}(z)w^{n-1} + ... + n_1(z)w + n_0(z), \quad (3.2a)$$
$$d(w,z) = \det H(w,z) = w^n - d_{n-1}(z)w^{n-1} - ... - d_1(z)w - d_0(z)$$

and

$$n_k(z) = b_k^0 + b_k^1 z^{-1} + ... + b_k^q z^{-q},$$
$$d_k(z) = a_k^0 + a_k^1 z^{-1} + ... + a_k^q z^{-q}, \quad (3.2b)$$
$$k = 0,1,..., n-1$$

the $H_{ad}(w,z)$ is the adjoint matrix.

From (3.1) we have

$$D = \lim_{w,z\to\infty} T(w,z) \quad (3.3)$$

$$\text{since} \lim_{w,z\to\infty}\left[I_n w - \sum_{r=0}^{q} A_{r\alpha} z^{-r}\right]^{-1} = 0.$$

The strictly proper transfer function is given by the equation

$$T_{sp}(w,z) = T(w,z) - D = \frac{N(w,z)}{d(w,z)} \quad (3.4)$$

and the realization problem is down to finding the matrices A, B and C.

Theorem 3.1. There exists a positive realization of the proper transfer function $T(w,z)$ of the fractional discrete-time linear system with delays (2.1) for $0<\alpha<1$ if the following conditions are satisfied:

1) $T(\infty,\infty)=\lim_{w,z\to\infty} T(w,z)\in\Re_+^{p\times m}$, which is equivalent to $D\in\Re_+^{p\times m}$.
2) Coefficients of the polynomial $d(w,z)$ are nonnegative, i.e. $a_k^r\geq 0$, $r = 0,1,\ldots q$; $k = 0,1,\ldots n-1$.
3) Coefficients of $N(w,z)$ are nonnegative, i.e. $b_k^r\geq 0$, $r = 0,1,\ldots q$; $k = 0,1,\ldots n-1$.

The realization have the following form

$$A_{0\alpha}=\begin{bmatrix}0 & \dots & 0 & a_0^0\\ 1 & \dots & 0 & a_1^0\\ \vdots & \ddots & \vdots & \vdots\\ 0 & \dots & 1 & a_{n-1}^0\end{bmatrix},\quad A_{r\alpha}=\begin{bmatrix}0 & \dots & 0 & a_0^r\\ 0 & \dots & 0 & a_1^r\\ \vdots & \dots & \vdots & \vdots\\ 0 & \dots & 0 & a_{n-1}^r\end{bmatrix}, r=1,\ldots q;$$
$$B_r=\begin{bmatrix}b_0^r\\ \vdots\\ b_{n-1}^r\end{bmatrix}, r=0,1,\ldots q;\quad C=[0 \ \ \dots \ \ 0 \ \ 1]. \tag{3.5}$$

Proof. According to (3.1) the strictly proper transfer function have the form

$$T_{sp}(w,z)=\frac{N(w,z)}{d(w,z)}$$
$$=\frac{C[I_n w - A_{0\alpha}-A_{1\alpha}z^{-1}-\dots-A_{q\alpha}z^{-q}]_{ad}[B_0+B_1z^{-1}+\dots+B_qz^{-q}]}{\det[I_n w - A_{0\alpha}-A_{1\alpha}z^{-1}-\dots-A_{q\alpha}z^{-q}]} \tag{3.6}$$

Developing the denominator $d(w,z)$ of the transfer function (3.6) according to n-th column and using the matrices (3.5) we obtain the following polynomial

$$d(w,z)=\det[I_n w - A_{0\alpha}-A_{1\alpha}z^{-1}-\dots-A_{q\alpha}z^{-q}]$$
$$=\det\begin{bmatrix} w & 0 & \dots & 0 & -(\sum_{i=0}^{q}a_0^1 z^{-i})\\ -1 & w & \dots & 0 & -(\sum_{i=0}^{q}a_1^1 z^{-i})\\ \vdots & \vdots & \dots & \vdots & \vdots\\ 0 & 0 & \dots & -1 & w-(\sum_{i=0}^{q}a_{n-1}^1 z^{-i})\end{bmatrix}$$
$$=w^n-(a_{n-1}^0+a_{n-1}^1z^{-1}+\dots+a_{n-1}^q z^{-q})w^{n-1} \tag{3.7}$$
$$-\dots-(a_1^0+a_1^1z^{-1}+\dots+a_1^q z^{-q})w-(a_0^0+a_0^1z^{-1}+\dots+a_0^q z^{-q})$$
$$=w^n-d_{n-1}w^{n-1}-\dots-d_1w-d_0$$

which is equal to (3.2).

It is well known [12] that if $A_{r\alpha}, r=0,1,\ldots q$ have the canonical form (3.5) then

$$C[I_n w - A_{0\alpha}-A_{1\alpha}z^{-1}-\dots-A_{q\alpha}z^{-q}]_{ad}=[1 \ \ w \ \ \dots \ \ w^{n-1}]. \tag{3.8}$$

Now developing the numerator $N(w,z)$ of the transfer function (3.6) we obtain the polynomial

$$N(w,z)=[1 \ \ w \ \ \dots \ \ w^{n-1}]\begin{bmatrix} b_0^0+b_0^1z^{-1}+\dots+b_0^q z^{-q}\\ b_1^0+b_1^1z^{-1}+\dots+b_1^q z^{-q}\\ \vdots\\ b_{n-1}^0+b_{n-1}^1z^{-1}+\dots+b_{n-1}^q z^{-q}\end{bmatrix} \tag{3.9}$$
$$=(b_{n-1}^0+b_{n-1}^1z^{-1}+\dots+b_{n-1}^q z^{-q})w^{n-1}$$
$$-\dots-(b_1^0+b_1^1z^{-1}+\dots+b_1^q z^{-q})w-(b_0^0+b_0^1z^{-1}+\dots+b_0^q z^{-q})$$
$$=n_{n-1}w^{n-1}+n_{n-2}w^{n-2}+\dots+n_1w+n_0.$$

which is equal to (3.2). This shows that the matrices (3.5) are realization of the strictly proper transfer function (3.4), moreover if the conditions of Theorem 3.1 are satisfied then the matrices (3.5) are positive realization. □

Based on Theorem 3.1 the following procedure for finding the realization of $T(w,z)$ is proposed.

Procedure.

Step 1. Compute matrix D and the strictly proper transfer function.

Step 2. Knowing coefficients a_k^r, $r = 0,1,\ldots q$; $k = 0,1,\ldots n-1$ of the polynomial $d(w,z)$ and the matrices (3.5) find $A_{r\alpha}, r=0,1,\ldots q$.

Step 3. Knowing coefficients b_k^r, $r = 0,1,\ldots q$; $k = 0,1,\ldots n-1$ of the $N(w,z)$ and the matrices (3.5) find $B_r, r=0,1,\ldots q$.

Step 4. Knowing fractional order α and (2.10b) we can find the realization of the fractional system (2.1).

Example 3.1. Find the positive realization of the discrete-time linear systems with delay and fractional order $\alpha=0.5$ given by the transfer function

$$T(w,z)=\frac{w^3-(z^{-1}-1)w^2-(z^{-1}-1)w-(z^{-1}+1)}{w^3-(z^{-1}+1)w^2-(z^{-1}+2)w-(2z^{-1}+1)}. \tag{3.10}$$

In his case $n = 3$, $p = 1$, $q = 1$. Following the procedure we have.

Step 1. Using (3.3) we obtain the matrix D

$$D=\lim_{w,z\to\infty}T(w,z)=[1] \tag{3.11}$$

and the strictly proper transfer function

$$T_{sp}(w,z)=T(w,z)-D$$
$$=\frac{2w^2+3w+3z^{-1}+2}{w^3-(z^{-1}+1)w^2-(z^{-1}+2)w-(2z^{-1}+1)} \tag{3.12}$$

Step 2. Denominator have the form

$$d(w,z)=w^3-(z^{-1}+1)w^2-(z^{-1}+2)w-(2z^{-1}+1)$$
$$=w^3-(a_2^1z^{-1}+a_2^0)w^2-(a_1^1z^{-1}+a_1^0)w-(a_0^1z^{-1}+a_0^0) \tag{3.13}$$

and the matrices $A_{0\alpha}, A_{1\alpha}$ have the form

$$A_{0\alpha}=\begin{bmatrix}0 & 0 & a_0^0\\ 1 & 0 & a_1^0\\ 0 & 1 & a_2^0\end{bmatrix}=\begin{bmatrix}0 & 0 & 1\\ 1 & 0 & 2\\ 0 & 1 & 1\end{bmatrix},$$
$$A_{1\alpha}=\begin{bmatrix}0 & 0 & a_0^1\\ 0 & 0 & a_1^1\\ 0 & 0 & a_2^1\end{bmatrix}=\begin{bmatrix}0 & 0 & 2\\ 0 & 0 & 1\\ 0 & 0 & 1\end{bmatrix}. \tag{3.14}$$

Step 3. Numerator have the form

$$N(w,z)=2w^2+3w+3z^{-1}+2$$
$$=(b_2^1z^{-1}+b_2^0)w^2+(bz^{-1}+b_1^0)w+(b_0^1z^{-1}+b_0^0) \tag{3.15}$$

the matrices B_0, B_1 have the form

$$B_0=\begin{bmatrix}b_0^0\\ b_1^0\\ b_2^0\end{bmatrix}=\begin{bmatrix}2\\ 3\\ 2\end{bmatrix},\quad B_1=\begin{bmatrix}b_0^1\\ b_1^1\\ b_2^1\end{bmatrix}=\begin{bmatrix}3\\ 0\\ 0\end{bmatrix} \tag{3.16}$$

and

$$C=[0 \ \ 0 \ \ 1]. \tag{3.17}$$

Step 4. From (2.10b) for $\alpha = 0.5$ we obtain

$$A_0 = A_{0\alpha} - I_3(-1)^2\binom{\alpha}{1} = A_{0\alpha} - I_3\alpha = \begin{bmatrix} -0.5 & 0 & 1 \\ 1 & -0.5 & 2 \\ 0 & 1 & 0.5 \end{bmatrix}$$

$$A_1 = A_{1\alpha} - I_3(-1)^3\binom{\alpha}{2} \tag{3.18}$$

$$= A_{1\alpha} + I_3\frac{\alpha(\alpha-1)}{2!} = \begin{bmatrix} -0.125 & 0 & 2 \\ 0 & -0.125 & 1 \\ 0 & 0 & 0.875 \end{bmatrix}$$

The realization of the fractional discrete-time linear system with delays is given by (3.18) (3.16) and (3.17), these matrices satisfy the Theorem 2.1 and they are the positive realization of the transfer function (3.10).

4. Concluding remarks

A method for computation of a positive realization of a given proper transfer matrix of fractional discrete-time linear systems with delays in state and input has been proposed. Sufficient conditions for the existence of a positive realization of this class of fractional systems have been established. A procedure for computation of a positive realization has been proposed. The effectiveness of the procedure has been illustrated by a numerical example. Extension of these considerations for multi-input multi-output fractional systems with delays is possible. An open problem is formulation of the necessary and sufficient conditions for the existence of positive minimal realizations for fractional.

5. Acknowledgment

This work was supported by National Centre of Science in Poland under work No. N N514 6389 40.

Bibliography

1. Benvenuti L., Farina L.: *A tutorial on the positive realization problem*, "IEEE Trans. Autom. Control", Vol. 49, No. 5, 2004, 651 664.
2. Engheta N., *On the role of fractional calculus in electromagnetic theory*, "IEEE Trans. Atenn. Prop.", Vol. 39, No. 4, 1997, 35 46.
3. Farina L., Rinaldi S.: *Positive Linear Systems*, Theory and Applications, J. Wiley, New York 2000.
4. Ferreira N.M.F, Machado J.A.T.: *Fractional-order hybrid control of robotic manipulators*, Proc. 11th Int. Conf. Advanced Robotics, ICAR '2003, Coimbra (Portugal) 2003, 393 398.
5. Kaczorek T.: *A realization problem for positive continues-time linear systems with reduced numbers of delay*, "Int. J. Appl. Math. Comp. Sci.", Vol. 16, No. 3, 2006, 325 331.
6. Kaczorek T.: *Computation of realizations of discrete-time cone systems*, "Bull. Pol. Acad. Sci. Techn.",
7. Kaczorek T.: *Fractional positive linear systems*, "Kybernetes: The International Journal of Systems & Cybernetics", Vol. 38, No. 7/8, 2009, 1059 1078.
8. Kaczorek T.: *Realization problem for fractional continuous-time systems*, Archives of Control Sciences, Vol. 18, No. 1, 2008, 43 58.
9. Kaczorek T.: *Realization problem for positive multivariable discrete-time linear systems with delays in the state vector and inputs*, "Int. J. Appl. Math. Comp. Sci.", Vol. 16, No. 2, 2006, 101 106.
10. Kaczorek T.: *Realization problem for positive discrete-time systems with delay*, "System Science", Vol. 30, No. 4, 2004, 117 130.
11. Kaczorek T.: *Realization problem for positive fractional discrete-time linear systems*, [in:] Pennacchio S. (ed.): *Emerging Technologies. Robotics and Control Systems*, Int. Society for Advanced Research, 2008, 226 236.
12. Kaczorek T.: *Positive 1D and 2D Systems*, Springer-Verlag, London 2002.
13. Kaczorek T.: *Positive minimal realizations for singular discrete-time systems with delays in state and delays in control*, "Bull. Pol. Acad. Sci. Techn.", Vol. 53, No. 3, 2005, 293 298.
14. Kaczorek T.: *Selected Problems in Fractional Systems Theory*, Springer-Verlag 2011.
15. Klamka J.: *Approximate constrained controllability of mechanical systems*, "Journal of Theoretical and Applied Mechanics", Vol. 43, No. 3, 2005, 539 554.
16. Miller K. S., Ross B.: *An Introduction to the Fractional Calculus and Fractional Differenctial Equations*, Willey, New York 1993.
17. Nishimoto K.: *Fractional Calculus*, Koriama: Decartess Press, 1984.
18. Oldham K. B., Spanier J.: *The Fractional Calculus*, Academic Press, New York 1974.
19. Ortigueira M. D.: *Fractional discrete-time linear systems*. Proc. of the IEE-ICASSP 97, Munich, Germany, IEEE, Vol. 3, New York 1997, 2241 2244.
20. Ostalczyk P.: *The non-integer difference of the discrete-time function and its application to the control system synthesis*, "Int. J. Syst, Sci.", Vol. 31, No. 12, 2000, 1551 1561.
21. Oustaloup A.: *Commande CRONE*, Hermés, Paris 1993.
22. Podlubny I.: *Fractional Differential Equations*. San Diego: Academic Press, 1999.
23. Podlubny I.: Dorcak L., Kostial I., *On fractional derivatives, fractional order systems and PIDμ-controllers*, Proc. 36th IEEE Conf. Decision and Control, San Diego, CA, 1997, 4985 4990.
24. Sajewski Ł.: *Realizacje dodatnie dyskretnych liniowych układów niecałkowitego rzędu w oparciu o odpowiedź impulsową*, „Measurement Automation and Monitoring", Vol. 56, No. 5, 2010, 404 408.

25. Zaborowsky V., Meylaov R.: *Informational network traffic model based on fractional calculus*. Proc. Int. Conf. Info-tech and Info-net, ICII 2001, Beijing, China, vol. 1, 2001, 58–63. ■

Dodatnia realizacja dla klasy dyskretnych układów liniowych z opóźnieniami

Streszczenie: Sformułowane zostanie zadanie realizacji dodatniej dla klasy dyskretnych układów liniowych niecałkowitego rzędu z opóźnieniami w wektorze stanu i wymuszeniu. Zaproponowana zostanie metoda wyznaczania realizacji dodatniej dla danej transmitancji właściwej układu o jednym wejściu i jednym wyjściu. Podane zostaną warunki wystarczające istnienia realizacji dodatniej dla rozpatrywanej klasy liniowych układów z opóźnieniami. Przedstawiona zostanie procedura wyznaczania realizacji dodatniej zilustrowana przykładem numerycznym.

Słowa kluczowe: niecałkowity rząd, dodatni, opóźnienia, realizacja, istnienie, wyznaczanie

dr inż. Łukasz Sajewski

Urodzony 8 grudnia 1981 roku w Białymstoku. Tytuł magistra inżyniera Elektrotechniki uzyskał w lipcu 2006 roku na Wydziale Elektrycznym Politechniki Białostockiej. Na tej samej uczelni w czerwcu 2009 roku, jako słuchacz 3 roku studiów doktoranckich obronił rozprawę doktorską otrzymując tytuł doktora nauk technicznych w dyscyplinie Elektrotechnika. W chwili obecnej pracuje na Wydziale Elektrycznym PB w Katedrze Automatyki i Elektroniki. Główny zakres jego zainteresowań naukowych to nowoczesna teoria sterowania, w tym układy dodatnie i hybrydowe (ciągło-dyskretne).

e-mail: lsajewski@pb.edu.pl