Approximation of positive stable continuous-time linear systems by positive stable discrete-time systems

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Abstract: The positive asymptotically stable continuous-time linear systems are approximated by positive asymptotically stable discrete-time linear systems by the use of Pade type approximation. It is shown that the approximation preserves the positivity and asymptotic stability of the systems. The stabilization problem of positive unstable continuous-time and corresponding discrete-time linear systems by state-feedbacks is also addressed.

Keywords: approximation, continuous-time, discrete-time, linear positive system, stability

1. Introduction

In positive systems inputs, state variables and outputs take only non-negative values. Examples of positive systems are industrial processes inVolving chemical reactors, heat exchangers and distillation columns, storage systems, compartmental systems, water and atmospheric pollution models. A variety of models having positive linear behavior can be found in engineering, management science, economics, social sciences, biology and medicine, etc. Positive linear systems are defined on cones and not on linear spaces. Therefore, the theory of positive systems is more complicated and less advanced. An overview of state of the art in positive systems theory is given in the monographs [5, 7].

Stability of positive linear systems has been investigated in [5, 7] and of fractional linear systems in [2-4, 10]. The problem of preservation of positivity by approximation the continuous-time linear systems by corresponding discrete-time linear systems has been addressed in [8].

In this paper it will be shown that using Pade type approximation of the exponential matrix the positive asymptotically stable continuous-time linear systems can be approximated by corresponding positive asymptotically stable discrete-time linear systems.

The paper is organized as follows. In section 2 basic definitions and theorems concerning positive continuous-time and discrete-time linear systems are recalled. The positivity of the linear systems are considered in section 3 and the asymptotic stability of the systems in section 4. The stabilization problem by state-feedbacks of the positive systems is addressed in section 5. Concluding remarks are given in section 6.

The following notation will be used: $\Re$ - the set of real numbers, $\Re^{n\times m}$ – the set of $n\times m$ real matrices, $\Re_+^{n\times m}$ – the set of $n\times m$ matrices with nonnegative entries and $\Re_+^n = \Re_+^{n\times 1}$, M_n – the set of $n\times n$ Metzler matrices (real matrices with nonnegative off-diagonal entries), M_{ns} – the set of $n\times n$ asymptotically stable Metzler matrices, $\Re_{+s}^{n\times n}$ – the set of $n\times n$ asymptotically stable positive matrices, I_n – the $n\times n$ identity matrix.

2. Preliminaries and the problem formulation

Consider the continuous-time linear system

$$\dot{x}(t) = A_c x(t) + B_c u(t), \quad x(0) = x_0 \tag{2.1}$$

where $x(t)\in\Re^n$, $u(t)\in\Re^m$ are the state and input vectors and $A_c\in\Re^{n\times n}$, $B_c\in\Re^{n\times m}$.

Definition 2.1. [5, 7] The system (2.1) is called (internally) positive if $x(t)\in\Re_+^n$, $t\ge 0$ for any initial conditions $x(0)=x_0\in\Re_+^n$ and all inputs $u(t)\in\Re_+^m$, $t\ge 0$.

Theorem 2.1. [5, 7] The system (2.1) is positive if and only if

$$A_c \in M_n, \quad B_c\in\Re_+^{n\times m}. \tag{2.2}$$

Definition 2.2. [5, 7] The positive system (2.1) is called asymptotically stable if for $u(t)=0$, $t\ge 0$

$$\lim_{t\to\infty} x(t) = 0 \text{ for all } x_0\in\Re_+^n. \tag{2.3}$$

Theorem 2.2. [5, 7] The positive system (2.1) is asymptotically stable if and only if all coefficients of the polynomial

$$\det[I_n s - A_c] = s^n + a_{n-1}s^{n-1} + \ldots + a_1 s + a_0 \tag{2.4}$$

are positive, i.e. $a_i > 0$ for $i = 0, 1, \ldots, n-1$.

Now let us consider the discrete-time linear system

$$x_{i+1} = A_d x_i + B_d u_i, \quad i\in Z_+ \tag{2.5}$$

where $x_i \in \Re^n$, $u_i \in \Re^m$ are the state and input vectors and $A_d \in \Re^{n\times n}$, $B_d \in \Re^{n\times m}$.

Definition 2.3. [5, 7] The system (2.5) is called (internally) positive if $x_i \in \Re_+^n$, $i \in Z_+$ for any initial conditions $x_0 \in \Re_+^n$ and all inputs $u_i \in \Re_+^m$, $i \in Z_+$.

Theorem 2.3. [5, 7] The system (2.5) is positive if and only if

$$A_d \in \Re_+^{n\times n},\ B_d \in \Re_+^{n\times m}. \tag{2.6}$$

Definition 2.4. [5, 7] The positive system (2.5) is called asymptotically stable if for $u_i = 0$, $i \in Z_+$

$$\lim_{i\to\infty} x_i = 0 \text{ for all } x_0 \in \Re_+^n. \tag{2.7}$$

Theorem 2.4. [5, 7] The positive system (2.5) is asymptotically stable if and only if all coefficients of the polynomial

$$\det[I_n(z+1) - A_d] = z^n + \bar{a}_{n-1}z^{n-1} + \ldots + \bar{a}_1 z + \bar{a}_0 \tag{2.8}$$

are positive, i.e. $\bar{a}_i > 0$ for $i = 0,1,\ldots,n-1$.

It is well-known that if the sampling is applied to the continuous-time system (2.1) then the corresponding discrete-time system (2.5) has the matrices

$$A_d = e^{A_c h},\ B_d = \int_0^h e^{A_c t} B_c dt \tag{2.9}$$

where $h > 0$ is the sampling time.

In this paper the following approximation of the matrix (2.9) will be applied

$$A_d = [A_c + I_n\alpha][I_n\alpha - A_c]^{-1} \tag{2.10}$$

where the coefficients $\alpha = \alpha(h) = \dfrac{\alpha}{h} > 0$ is chosen so that $[A_c + I_n\alpha] \in \Re_+^{n\times n}$. It is well-known [1] that if $A_c \in M_{ns}$ then $\det[I_n\alpha - A_c] \neq 0$ for any $\alpha > 0$.

In the next sections it will be shown that the approximation (2.10) preserves:

1. the positivity, i.e. if $A_c \in M_n$ then $A_d \in \Re_+^{n\times n}$,
2. the asymptotic stability, i.e. if $A_c \in M_{ns}$ then $A_d \in \Re_{+s}^{n\times n}$.

3. Positivity of the systems

In what follows the following lemma will be used.

Lemma 3.1. If $A_n \in M_{ns}$ then

$$-A_n^{-1} \in \Re_+^{n\times n}. \tag{3.1}$$

Proof. The proof will be accomplished by induction. For $n = 1$ the hypothesis is evident. The hypothesis is true for $n = 2$ since

$$-A_2^{-1} = \begin{bmatrix} a_{11} & -a_{12} \\ -a_{21} & a_{22} \end{bmatrix}^{-1} = \frac{1}{a_{11}a_{22} - a_{12}a_{21}} \begin{bmatrix} a_{22} & a_{12} \\ a_{21} & a_{11} \end{bmatrix} \in \Re_+^{2\times 2} \tag{3.2}$$

for $a_{i,j} \geq 0$; $i, j = 1,2$.

Assuming that the hypothesis is true for $k \geq 1$ we shall show that it is also valid for $k+1$. Let $A_{k+1} \in M_{k+1}$, $\det A_{k+!} \neq 0$ and

$$-A_{k+1} = \begin{bmatrix} a_{11} & -a_{12} & \ldots & -a_{1,k+1} \\ -a_{21} & a_{22} & \ldots & -a_{2,k+1} \\ \vdots & \vdots & \ddots & \vdots \\ -a_{k+1,1} & -a_{k+1,2} & \ldots & a_{k+1,k+1} \end{bmatrix},$$
$$a_{i,j} \geq 0;\ i, j = 1,2,\ldots,k+1 \tag{3.3}$$

then it is well-known [9] that

$$-A_{k+1}^{-1} = \begin{bmatrix} -A_k^{-1} + \dfrac{A_k^{-1}u_k v_k A_k^{-1}}{a_{k+1}} & \dfrac{A_k^{-1}u_k}{a_{k+1}} \\ \dfrac{v_k A_k^{-1}}{a_{k+1}} & \dfrac{1}{a_{k+1}} \end{bmatrix} \tag{3.4}$$

where

$$-A_k = \begin{bmatrix} a_{11} & -a_{12} & \ldots & -a_{1,k} \\ -a_{21} & a_{22} & \ldots & -a_{2,k} \\ \vdots & \vdots & \ddots & \vdots \\ -a_{k,1} & -a_{k,2} & \ldots & a_{k,k} \end{bmatrix} \in M_k,$$
$$a_{i,j} \geq 0;\ i, j = 1,2,\ldots,k,$$
$$v_k = [-a_{k+1,1}\ \ -a_{k+1,2}\ \ \ldots\ \ -a_{k+1,k}],$$
$$u_k^T = [-a_{1,k+1}\ \ -a_{2,k+1}\ \ \ldots\ \ -a_{k,k+1}],$$
$$a_{k,k} = a_{k+1,k+1} - v_k A_k^{-1} u_k > 0. \tag{3.5}$$

By assumption $-A_k^{-1} \in \Re_+^{k\times k}$ and $-v_k^T \in \Re_+^k$, $-v_k \in \Re_+^k$, $a_{k+1} > 0$. Hence from (3.4) we have $-A_{k+1}^{-1} \in \Re_+^{k+1\times k+1}$. This completes the proof. □

Theorem 3.1. If the continuous-time system (2.1) is positive and asymptotically stable then the discrete-time system (2.5) with the matrix (2.10) is also positive for any sampling time $h > 0$.

Proof. If the continuous-time system (2.1) is positive and asymptotically stable then $A_c \in M_{ns}$ and there exists such $\alpha > 0$ that $[A_c + I_n\alpha] \in \Re_+^{n\times n}$. If $A_c \in M_{ns}$ then $\det[I_n\alpha - A_c] \neq 0$ for any $\alpha > 0$ and $[I_n\alpha - A_c]^{-1} \in \Re_+^{n\times n}$. In this case $A_d = [A_c + I_n\alpha][I_n\alpha - A_c]^{-1} \in \Re_+^{n\times n}$ and the discrete-time system (2.5) by Theorem 2.3 is positive. □

4. Asymptotic stability of the system

Lemma 4.1. If s_k $k = 1, 2, ..., n$ are eigenvalues of the matrix $A_c \in M_n$ then the eigenvalues z_k $k = 1,2,...,n$ of the matrix $A_d \in \Re_+^{n\times n}$ defined by (2.10) are given by

$$z_k = \frac{s_k + \alpha}{\alpha - s_k} \quad \text{for } k = 1,2,...,n \tag{4.1}$$

Proof. If $A_c \in M_n$, $\alpha > 0$ is chosen so that $[A_c + I_n\alpha] \in \Re_+^{n\times n}$ and $\alpha \neq s_k$ then the function $f(s_k) = \frac{s_k + \alpha}{\alpha - s_k}$ is well defined on the spectrum s_k $k = 1,2,...,n$ of the matrix A_c. In this case it is well-known [6, 9] that equality (4.1) holds. □

Theorem 4.1. If the positive continuous-time system (2.1) is asymptotically stable then the corresponding discrete-time positive system (2.5) is also asymptotically stable.

Proof. If the positive continuous-time system (2.1) is asymptotically stable then the real parts $-\alpha_k$ of its eigenvalues $s_k = -\alpha_k \pm j\beta_k$, $k = 1,2,...,n$ are negative. In this case using (4.1) we obtain

$$|z_k| = \left|\frac{\alpha - \alpha_k \pm j\beta_k}{\alpha + \alpha_k \mp j\beta_k}\right| = \frac{|\alpha - \alpha_k \pm j\beta_k|}{|\alpha + \alpha_k \mp j\beta_k|} < 1 \tag{4.2}$$

and the discrete-time system (2.5) is also asymptotically stable. □

5. Stabilization of the system

Consider the positive continuous-time linear system (2.1) and the corresponding positive discrete-time linear system (2.5). It is assumed that

$$\det A_c \neq 0 \text{ and } \operatorname{rank} B_c = m . \tag{5.1}$$

If $\det A_c \neq 0$ then from (2.9) we have

$$B_d = A_c^{-1}(e^{A_c h} - I_n)B_c \tag{5.2}$$

and

$$\operatorname{rank} B_d = m \tag{5.3}$$

since $\det[e^{A_c h} - I_n] \neq 0$ and $\operatorname{rank} B_c = m$.

If the positive system (2.1) is unstable then applying a suitable state-feedback with a matrix $K_c \in \Re^{m\times n}$ we may stabilize the system, i.e.

$$\overline{A}_c = A_c + B_c K_c \in M_{ns} . \tag{5.4}$$

The corresponding matrix of the discrete-time close-loop system

$$\overline{A}_d = [\overline{A}_c + I_n\alpha][I_n\alpha - \overline{A}_c]^{-1} \in \Re_+^{n\times n} \tag{5.5}$$

is nonnegative and asymptotically stable.

By Theorem 4.1 if s_k $k = 1, 2, ..., n$ are the eigenvalues of $\overline{A}_c$, located in the open left half of the complex plane, then the eigenvalues z_k $k = 1, 2, ..., n$ of $\overline{A}_d$ are given by (4.1) and are located in the unit circle of the complex plane. Therefore, the asymptotic stability of the continuous-time system with $\overline{A}_c$ implies the asymptotic stability of the discrete-time system with $\overline{A}_d$ defined by (5.5).

Let the discrete-time system with A_d be unstable. We are looking for a state-feedback matrix $K_d \in \Re^{m\times n}$ such that the close-loop system is positive and asymptotically stable with the matrix $\overline{A}_d$, i.e.

$$\overline{A}_d = A_d + B_d K_d \in \Re_{+s}^{n\times n} . \tag{5.6}$$

Solving the equation (5.6) with respect to K_d for given $\overline{A}_d$, A_d and B_d we obtain

$$K_d = [B_d^T B_d]^{-1} B_d^T [\overline{A}_d - A_d] . \tag{5.7}$$

The matrix (5.7) is the solution of (5.6) if and only if

$$B_d [B_d^T B_d]^{-1} B_d^T [\overline{A}_d - A_d] = \overline{A}_d - A_d . \tag{5.8}$$

Therefore, the following theorem has been proved.

Theorem 5.1. There exists a state-feedback gain matrix (5.7) of the positive and asymptotically stable discrete-time close-loop system if the condition (5.8) is met.

Remark 5.1. The state-feedback gain matrix K_c and K_d stabilizing the systems are in general case different and are related by

$$\begin{aligned} &[A_c + I_n\alpha + B_c K_c][I_n\alpha - A_c - B_c K_c]^{-1} \\ &\quad = [A_c + I_n\alpha][I_n\alpha - A_c]^{-1} \\ &\quad + A_c^{-1}\{[A_c + I_n\alpha][I_n\alpha - A_c]^{-1} - I_n\} B_c K_d . \end{aligned} \tag{5.9}$$

This equality follows immediately from (5.5), (5.4), (5.2) and (2.10).

Example 5.1. Given the positive unstable continuous-time system (2.1) with the matrices

$$A_c = \begin{bmatrix} -2 & 1 \\ 1 & 1 \end{bmatrix}, \quad B_c = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{5.10}$$

Find a state-feedback gain matrix $K_c \in \Re^{1\times 2}$ which preserve the positivity and stabilize the system.
Let the close-loop matrix has the form

$$\overline{A}_c = \begin{bmatrix} -2 & 1 \\ 0 & -3 \end{bmatrix}. \tag{5.11}$$

In this case the state-feedback gain matrix has the form

$$K_c = [-1 \quad -4] \tag{5.12}$$

since

$$\overline{A}_c = A_c + B_c K_c = \begin{bmatrix} -2 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}[-1 \quad -4] = \begin{bmatrix} -2 & 1 \\ 0 & -3 \end{bmatrix}. \tag{5.13}$$

Using (2.10) and (5.2) we can compute the matrices A_d and B_d of the corresponding discrete-time system (2.5) for $h = 1$ and $\alpha = 4$

$$A_d = [A_c + I_n\alpha][I_n\alpha - A_c]^{-1} = \begin{bmatrix} 2 & 1 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} 6 & -1 \\ -1 & 3 \end{bmatrix}^{-1} = \frac{1}{17}\begin{bmatrix} 7 & 8 \\ 8 & 31 \end{bmatrix} \tag{5.14}$$

and

$$B_d = A_c^{-1}[A_d - I_n]B_c = \begin{bmatrix} -2 & 1 \\ 1 & 1 \end{bmatrix}\frac{1}{17}\begin{bmatrix} -10 & 8 \\ 8 & 14 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{17}\begin{bmatrix} 2 \\ 12 \end{bmatrix}. \tag{5.15}$$

By Theorem 2.3 the discrete-time system is positive since the matrices (5.14) and (5.15) have positive entries but the system is unstable. The polynomial (2.8) for the matrix (5.14) has the form

$$\det[I_n(z+1) - A_d] = \begin{vmatrix} z + \frac{10}{17} & -\frac{8}{17} \\ -\frac{8}{17} & z - \frac{14}{17} \end{vmatrix} = z^2 - \frac{4}{17}z - \frac{204}{289}. \tag{5.16}$$

By Theorem 2.4 the discrete-time system is unstable since two coefficients of the polynomial (5.16) are negative.
Using (4.1) and taking into account that the matrix $\overline{A}_c$ has the eigenvalues $s_1 = -2$, $s_2 = -3$ we obtain

$$z_1 = \frac{s_1 + \alpha}{\alpha - s_1} = \frac{-2+4}{4+2} = \frac{1}{3}, \quad z_2 = \frac{s_2 + \alpha}{\alpha - s_2} = \frac{-3+4}{4+3} = \frac{1}{7} \tag{5.17}$$

Therefore, the corresponding close-loop discrete-time system is also asymptotically stable.
Using (2.10) we may compute the matrix $\overline{A}_d$ of the close-loop system

$$\overline{A}_d = [\overline{A}_c + I_n\alpha][I_n\alpha - \overline{A}_c]^{-1} = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 6 & -1 \\ 0 & 7 \end{bmatrix}^{-1} = \frac{1}{21}\begin{bmatrix} 7 & 4 \\ 0 & 3 \end{bmatrix} \tag{5.18a}$$

and

$$\overline{B}_d = \overline{A}_c^{-1}[\overline{A}_d - I_n]B_c = \begin{bmatrix} -2 & 1 \\ 0 & -3 \end{bmatrix}^{-1}\left\{\frac{1}{21}\begin{bmatrix} 7 & 4 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right\}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{21}\begin{bmatrix} 1 \\ 6 \end{bmatrix}. \tag{5.18b}$$

Figure 1 presents step response of the continuous-time system with matrices $\overline{A}_c$ and B_c and its discrete-time approximation with matrices (5.18).

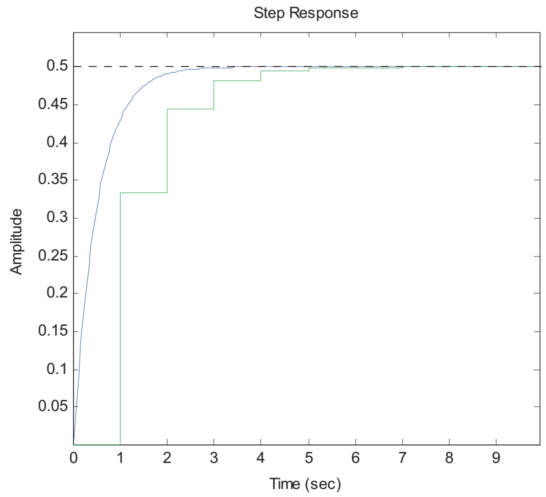

Fig. 1. Step response of the continuous-time system and its discrete-time approximation
Rys. 1. Odpowiedź skokowa układu z czasem ciągłym i jej aproksymacja dyskretno-czasowa

Next from (5.7) the state-feedback gain matrix

$$K_d = [B_d^T B_d]^{-1} B_d^T [\overline{A}_d - A_d] = \left[\frac{1}{289}[2 \quad 12]\begin{bmatrix} 2 \\ 12 \end{bmatrix}\right]^{-1}\frac{1}{17}[2 \quad 12]\left(\frac{1}{21}\begin{bmatrix} 7 & 4 \\ 0 & 3 \end{bmatrix} - \frac{1}{17}\begin{bmatrix} 7 & 8 \\ 8 & 31 \end{bmatrix}\right) = [-0.3414 \quad -1.2193]. \tag{5.19}$$

Note that the matrix (5.19) is different then the matrix (5.12).

Using (2.9) we may compute the matrix $\tilde{A}_d$ of the close-loop system

$$\tilde{A}_d = e^{\overline{A}_c h} = \begin{bmatrix} 0.1353 & 0.0855 \\ 0 & 0.0498 \end{bmatrix} \tag{5.20a}$$

and

$$\widetilde{B}_d = \overline{A}_c^{-1}[e^{\overline{A}_c h} - I_n]B_c \quad = \begin{bmatrix} -2 & 1 \\ 0 & -3 \end{bmatrix}^{-1}$$
$$\times \left\{ \begin{bmatrix} 0.1353 & 0.0855 \\ 0 & 0.0498 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.1156 \\ 0.3167 \end{bmatrix}. \quad (5.20b)$$

In figure 2 we have the same step response of the continuous-time system but with discrete-time system given by the matrices (5.20).

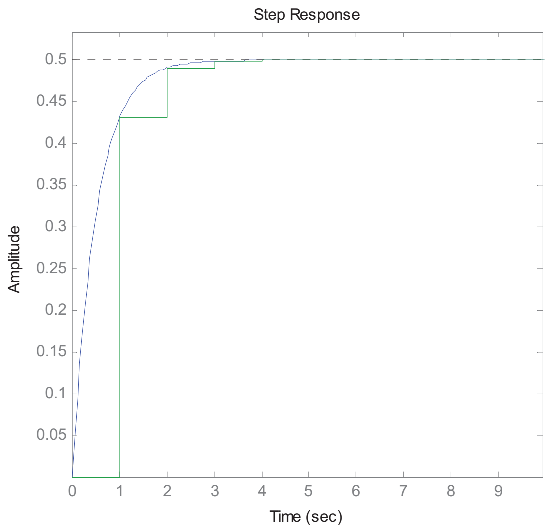

Fig. 2. Step response of the continuous-time system and its discrete representation (5.20)

Rys. 2. Odpowiedź skokowa układu z czasem ciągłym I jej dyskretna reprezentacja (5.20)

6. Concluding remarks

The approximation of positive asymptotically stable continuous-time linear system by the use of Pade type approximation of the exponential matrix has been addressed. It has been shown that the approximation preserves the positivity and asymptotic stability of the systems. The stabilization problem of unstable positive linear system by state-feedback has been analyzed. Sufficient conditions for the stabilization of discrete-time linear systems by state-feedbacks have been established. The considerations have been illustrated by numerical example. The presented approach can be extended for fractional linear systems [10].

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Aproksymacja dodatnich stabilnych ciągłych układów liniowych przez dodatnie stabilne układy dyskretne

Streszczenie: Dodatnie układy stabilne ciągłe są aproksymowane za pomocą liniowej aproksymacji Pade dodatnimi, stabilnymi układami dyskretnymi. Wykazano, że aproksymacja ta zachowuje dodatniość i stabilność asymptotyczną. Rozważania ogólne zostały zilustrowane przykładem numerycznym.

Słowa kluczowe: aproksymacja, układ ciągły, dyskretny, dodatni, stabilny

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