Benefits of Use of Spur Gears with Asymmetric Profile

Jarmila Vojtková
Technical University of Košice, Faculty of Mechanical Engineering, Letná 9, 040-01 Košice, Slovakia

Abstract: Using of different angles for the left and right side allows creating a tooth with the beneficial parameters. The software was developed to control the shape of the teeth, for calculation of the reduced radii of curvature of the asymmetrical teeth, Hertz pressures, the pressure angles limits and specific sliding. The asymmetrical gearing is suitable mainly for one direction of rotation, and the pinion can, therefore, have a lower number of teeth. By comparing the values, one can determine the benefits of their application.

Keywords: involute curve, asymmetric tooth, specific sliding, pressure angle

1. Introduction

Symmetrical teeth are commonly used for the entire spectrum of applications [3. 4]. Classical design of gears is based on the number of teeth and design module, the shape of basic rack is defined by the standard module and the pressure equal to 20°. The minimum number of teeth, for the normalized pressure angle, also depends on the addendum coefficient value. To improve the mesh conditions, various tooth corrections are mainly used. The most common type of correction is a correction by moving the tooth profile, and in case of helical gears, it is also a correction of helical angle. The majority of corrections are applied in order to achieve not only the desired centre-to-centre distance, but also to improve strength, stiffness properties of the teeth, or to remove the teeth undercutting or balance the specific sliding [1-3, 7, 9].

2. Asymmetric Teeth

Involute curve changes its shape depending on the pressure angle (Fig. 1).

In Fig. 1, there are involute curves for pinions with a number of the teeth $z_1 = 17$, module 10 mm and selected angles α are 15°, 20°, 30°, 40°.

An asymmetric involute tooth is formed by two involute curves. The involute curves satisfy the law of gearing. The mesh is only on the line of action. The tooth asymmetry is created by different values of the tooth pressure angle for the left side α_L and the right side α_P [1–3, 6, 7].

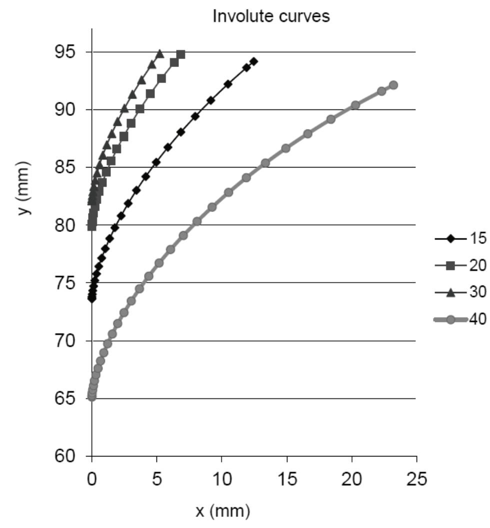

Fig. 1. Involute curves for a variety of pressure angles
Rys. 1. Ewolwentowe profile zęba dla różnych kątów przyporu

Pictures in the article are made so that the left side is determined from the minimum number of teeth (i.e. defined by the angle α_L). The right side is determined by the condition of the maximum value of the angle α_P, taking into account that the total top land thickness is $\geq 0.25\ m_n$. At a fixed value of the left pressure angle, the right pressure angle can be from the interval $<\alpha_L, \alpha_P>$. The right involute can be between the curves 1 and 2 (Fig. 2b). As shown in Fig. 2c, it is possible to see a change relative to the symmetrical shape, the area 3 is removed and the area 4, is added to the symmetrical shape. If, however, the left angle α_L is increased, the right angle α_P is decreased in order to comply with the condition of a sufficient top tooth thickness [1–3]. The limit values of pressure angles can be determined by using the custom made software (Fig. 5). Figure 3 presents a tooth segment with a large asymmetry, the number of the teeth of pinion $z_1 = 17$. Base circle is not identical for left and right sides. The base circle diameter is smaller for a bigger angle α.

Autor korespondujący:
Jarmila Vojtková, jarmila.vojtkova@tuke.sk
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In Fig. 4, there are the mesh and radii of curvature for a different direction of rotation. Points N_1 and N_2 are the interference points. As the radius of curvature increases, the pressure angle α increases also. The change of angle α leads to changes in the radii of curvature, which affects the Hertz pressures. Centre-to-centre distance stays the same.

The minimum number of teeth with allowed undercutting z'_{min}

$$z'_{\min} = \frac{5}{6} \cdot \frac{2 \cdot h_a^*}{\sin^2 \alpha} \quad (1)$$

where: h_a^* – tooth addendum coefficient, α – pressure angle (°).

The radii of curvature at the first point of mesh A (Fig. 4) for the tooth defined by pressure angle α_L and α_P are:

$$\rho_{1AL} = (a \cdot \sin \alpha_L) - \sqrt{(r_2 + h_a^* \cdot m)^2 - (r_2 \cdot \cos \alpha_L)^2} \quad (2)$$

$$\rho_{1AP} = (a \cdot \sin \alpha_P) - \sqrt{(r_2 + h_a^* \cdot m)^2 - (r_2 \cdot \cos \alpha_P)^2} \quad (3)$$

where: m – module (mm), r_2 – pitch radius of the wheel (mm), a – centre distance (mm).

Specific sliding of pinion for the left side is:

$$\vartheta_{1AL} = \frac{\rho_{1AL} \cdot \omega_1 - \rho_{2AL} \cdot \omega_2}{\rho_{1AL} \cdot \omega_1} \quad (4)$$

where: ϑ_{1AL} – specific sliding at the first point A of the mesh, α_L– pressure angle (°), ω_1 – input angular velocity (rad/s), ω_2 – output angular velocity (rad/s), ρ_{1AL} – radius of curvature at the point A.

A formula for specific sliding at point A for the right side is analogical to formula (4).

3. Software for Control and Analysis

The software (Fig. 5, Fig. 6) was developed to address the issue with asymmetric teeth, it allows to check and compare the following important parameters:

- design of a pinion without undercutting,
- design of a pinion with allowed undercutting,
- contact ratio,
- calculation of the radii of curvature and their reduced values, i.e. basic characteristics of the calculation of Hertz pressures,
- pressure angles limits,
- specific sliding for pinions.

For asymmetric teeth, it is required to follow more parameters simultaneously. For given parameters, several separate profiles can be created.

The software has a number of work modes. In every work mode, a different number of parameters can be defined.

The software draws graphs for pressure angle limits (Fig. 5). It draws graphs for specific sliding separately for right and left sides (Fig. 6). It controls the value of contact ratio (must be larger than one). It controls sufficient top land thickness. It also calculates values of radii of curvature at different mesh points and draws their reduced values. The software checks possible interferences.

The important benefit of asymmetric tooth profile applications is mainly a significant reduction in specific sliding

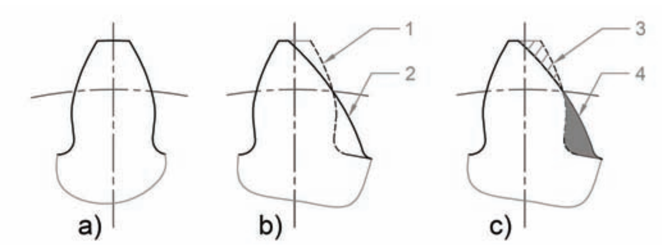

Fig. 2. Tooth shape: a) symmetrical tooth, b) asymmetric tooth, c) 3 – Area where material is removed, 4 – area where material is added
Rys. 2. Kształt zęba: a) ząb symetryczny, b) ząb asymetryczny, c) 3 – strefa z usuniętym materiałem, 4 – strefa z dodanym materiałem

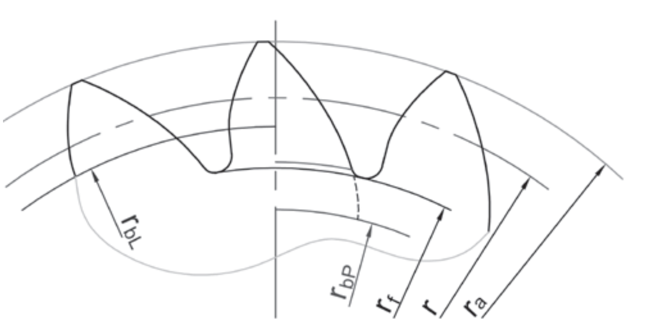

Fig. 3. Segment of the pinion, z_1= 17, α_L = 20°, α_P = 39.5°
Rys. 3. Segment wałka zębatego, z_1= 17, α_L = 20°, α_P = 39,5°

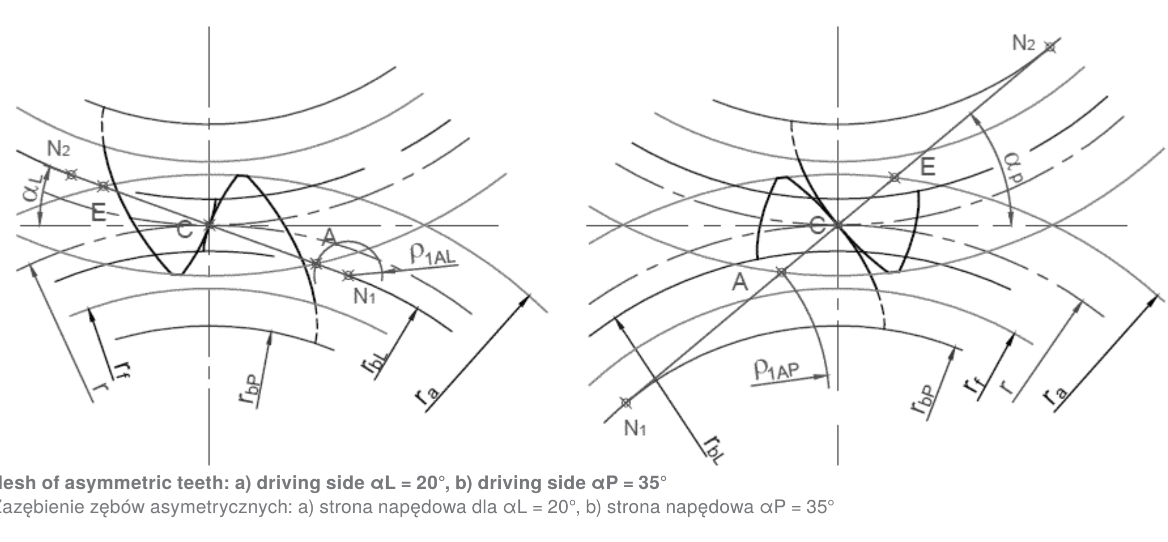

Fig. 4. Mesh of asymmetric teeth: a) driving side αL = 20°, b) driving side αP = 35°
Rys. 4. Zazębienie zębów asymetrycznych: a) strona napędowa dla αL = 20°, b) strona napędowa αP = 35°

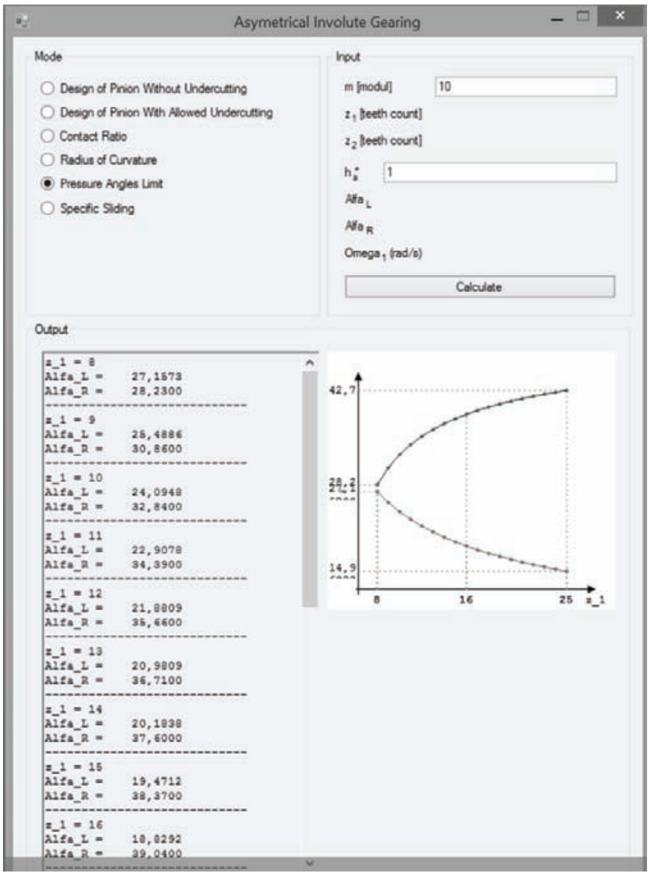

Fig. 5. The software for the control and analysis of asymmetric teeth, pressure angles limits for the left side and right side
Rys. 5. Oprogramowanie do sprawdzania i analizy zębów asymetrycznych, kątów granicznych przyporu dla strony lewej i prawej

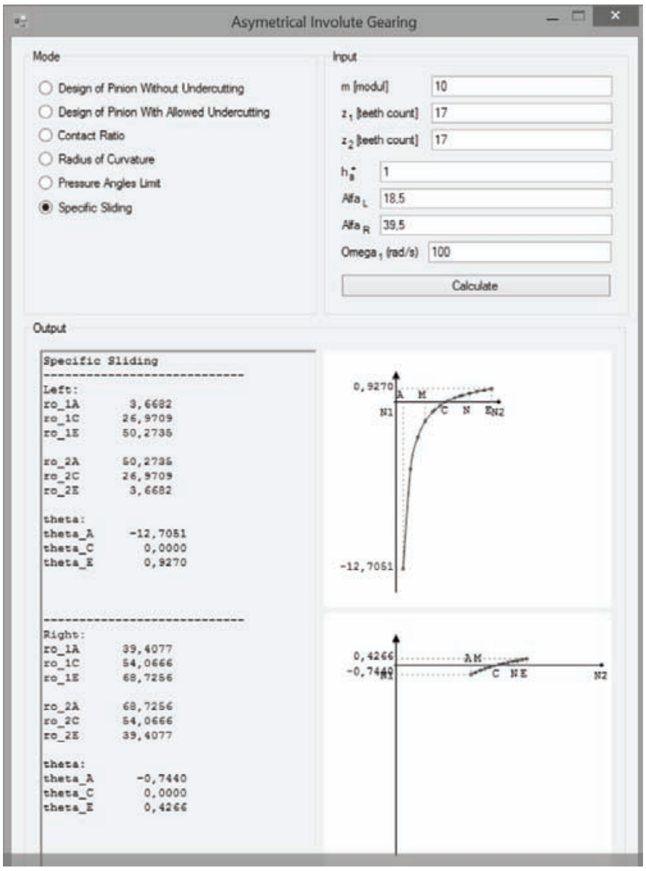

Fig. 6. The course of specific sliding, above, for driving side α_L=18.5°, below, for driving side α_P=39.5°
Rys. 6. Przebieg średnich poślizgów, u góry dla strony napędowej o kącie α_L=18,5 , u dołu dla strony napędowej o kącie α_P=39,5°

and Hertz pressures at the point of the first contact of the teeth. Fig. 7, specific sliding is a function of the pressure angle α. The left side is defined by standard value of 20°.

In the Tab. 1 the values for examined parameters by different number of the teeth z_1 are listed.

Tab. 1. Specific sliding and radii of curvature for gear ratio equal to 1 one and h_a^*=1
Tab. 1. Średnie poślizgi i promienie krzywizny dla przełożenia równego 1 i współczynnika h_a^*= 1

z_1	α_L(°)	ϑ_{1AL}	ρ_{1AL} (mm)	α_P(°)	ϑ_{1AP}	ρ_{1AL} (mm)
8	27	–27.03	1.25	28	–15.35	2.17
11	22	–43.70	0.90	34	–2.05	15.19
14	20	–18.37	2.35	37.5	–1.08	27.64
17	18	–17.95	2.63	39.5	–0.74	39.41
20	17	–12.21	4.12	40.5	–0.58	50.41

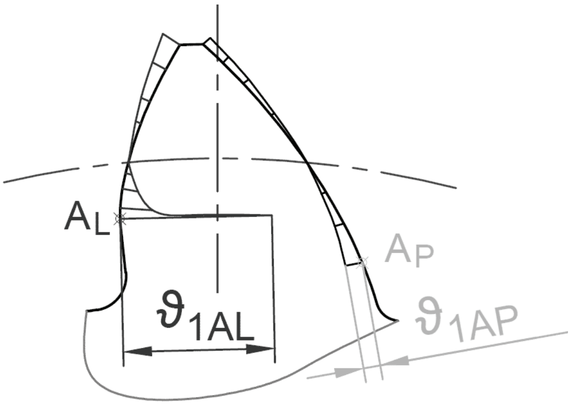

Fig. 7. The course of specific sliding on the involute tooth profile with the first point of mesh A defined for z_1=17, α_L=20°, α_P=39.5°
Rys. 7. Przebieg zmian poślizgu na profilu ewolwentowym zęba dla pierwszego punktu zazębienia A określonego przez z_1= 17, α_L=20°, α_P=39,5°

4. Conclusion

The asymmetry allows to decrease number of the teeth on the pinion. The assymmetric teeth make it possible to decrease the gearing dimensions and to reduce weight. The course of specific sliding is advantageous especially for larger pressure angles. However, a disadvantage may be a lower value of contact ratio.

The software mentioned herein allows quick and comfortable monitoring of asymmetric teeth. This gearing may be used only if the cost is not the ultimate criterion.

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Zalety stosowania czołowych kół zębatych o profilu asymetrycznym

Streszczenie: Zastosowanie różnych kątów dla lewej i prawej strony zęba pozwala na stworzenie kształtu o odpowiednich parametrach. Opracowane oprogramowanie umożliwia: ustalić właściwy kształt zęba oraz obliczyć zredukowane promienie krzywizny dla zębów asymetrycznych, naciski Herza, kąty graniczne profili oraz średnie poślizgi. Porównanie tych wartości pozwala określić zalety stosowania tego rodzaju profili asymetrycznych.

Słowa kluczowe: ewolwenta, ząb asymetryczny, specyficzne przesuwne, kąt ciśnienia

Ing. Jarmila Vojtková, PhD
jarmila.vojtkova@tuke.sk

Graduated in 1982 from Technical University in Košice – specialised in the field of Transport Machinery and Operation Equipment. Doctor of Engineering – expert in Parts of Machines. In her scientific work, she solves the issues of special gearing, toroidal drives and asymmetrical teeth. Author of 70 publications.