Numerical Analysis of Stability of the Variable Parameters, Fractional Order Transfer Functions

Krzysztof Oprzędkiewicz
Dept of Automatic Control and Robotics, Faculty of Electrical Engineering, Automatic Control, Informatics and Biomedical Engineering, AGH University, al. A. Mickiewicza 30, 30-059 Krakow, Poland

Abstract: In the paper the numerical analysis of the practical stability of the new, variable parameters, fractional order transfer function models is addressed. The considered transfer functions are constructed using discrete, variable parameters FOBD approximation. This implies that only sufficient unstability or equivalently necessary stability conditions can be given. They do not assure full and unique information about stability. The considered algorithm uses interval approach and Monte Carlo method. The application of interval analysis allows to quickly detect unstability at vertices of tested area. The proposed algorithm allows to check the stability of predefined range of parameters as well as to estimate of stability areas.

Keywords: fractional order transfer function, time-variant system, FOBD approximation, stability, Monte Carlo method

Acronyms

FO	– Fractional Order,
C definition	– Caputo definition,
GL definition	– Grünwald-Letnikov definition,
FOBD	– Fractional Order Backward Difference,
FOPID	– Fractional Order PID,
RL definition	– Riemann-Liouville definition,
VO	– Variable Order,
VT	– Variable Time Constant,
VOVT	– Variable Order and Variable Time Constant.

1. Introduction

Variable parameter and variable fractional order models are expected to precisely describe a behavior of physical systems or phenomena, for which the classic, constant parameter model is not enough. VO FOPID controllers are also required to assure better control performance as FOPID controllers with fixed orders.

The first proposal of a VO fractional derivative was developed by [1]. Their definition was based on the VO RL integral and the Marchaud derivative. Later [2] give definitions of VO derivative for RL, C and GL definitions as well as theoretical background for VO. In addition, this paper proposes the use of Fuzzy Logic to approximate of VO.

Various approximation methods for variable fractional order systems have been known since years. For example, the time--continuous approximation of VO system using C definition is proposed by [3]. An alternative approach to VO operator, using Scarpi integral and derivative is proposed by [4].

An estimation of variable parameters fractional system using Triple Estimation algorithm is presented by [5], the analytical solution of VO differential equations has been proposed by [6]. The solution of variable fractional order state equation with distributed delays is given by [7]. In [8] stability conditions for variable fractional dynamic systems especially dedicated to chaotic systems are proposed.

State of the art in the area of VO fractional operator has been given by [9]. It covers problems from theoretical background to various applications of VO operator. Applications in special issues of mechanics (e.g. viscoelastics), transport processes and anomalous diffusion, fractals, VO filters are recalled as well as applications of VO FOPID controller and other control algorithms employing VO operator.

The papers [10] and [11] propose the discrete approximation using GL definition and FOBD approximation as well as its application to construct variable fractional order inertial and oscillatory transfer functions. In these papers the time variable fractional order was included to FOBD approximator factors. This approach has been continued in the article [12], proposing the use of variable fractional order in the FOPID controller.

Variable parameters fractional systems with time-invariant order need to be mentioned too. Latest research in this area focuses mainly on control problems. For example [13] deals with time optimal control, [14–17] analyse the controllability and observability of these systems. The paper [18] proposes the extension of the Floquet-Lyapunov transformation to the fractional discrete-time linear systems with periodically varying parameters.

Autor korespondujący:
Krzysztof Oprzędkiewicz, kop@agh.edu.pl

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The new, VOVT inertial transfer function model is proposed in the paper [19]. This paper analyses its main properties, but the stability conditions can only be given for specific cases. In addition, they have a form of sufficient or necessary conditions. This implies that they are not able to give full information about stability.

In such a situation, a precise numerical analysis can be helpful. It is expected to check the stability of model for given ranges of its parameters (independently on their variability) as well as to estimate a range of stable parameters for given sample time of model.

In this paper the new, numerical method of stability analysis of the VOVT inertial transfer function is addressed. The proposed algorithm uses interval approach and Monte Carlo method.

The paper is organized as follows. Preliminaries draw theoretical background to present of main results. Next the proposed VOVT transfer functions are recalled. Furthermore the numerical method of stability testing is proposed and validated.

2. Preliminaries

2.1. Basics of fractional calculus

Elementary ideas from fractional calculus can be found in many books, e.g. [20–23]. Here only some definitions necessary to present of main results will be recalled.

First of all the fractional-order, integro-differential operator (see e.g. [20, 23, 24]) needs to be given. It is as follows:

Definition 1. *The elementary fractional order operator. The fractional-order integro-differential operator is defined as follows*:

$$ {}_{t_s}D^{\alpha}_{t_f} f(t) = \begin{cases} \dfrac{d^{\alpha} f(t)}{dt^{\alpha}} & \alpha > 0 \\ f(t) & \alpha = 0 \\ \displaystyle\int_{t_s}^{t_f} f(\tau)(d\tau)^{\alpha} & \alpha < 0 \end{cases} \tag{1} $$

where t_s and t_f denote time limits for operator calculation, $\alpha \in \mathbb{R}$ denotes the non-integer order of the operation.
Next recall the Gamma Euler function given e.g. by [24]:

Definition 2. *The Gamma function*

$$ \Gamma(x) = \int_0^{\infty} t^{x-1} e^{-t} dt. \tag{2} $$

A Mittag-Leffler function is a non-integer order generalization of the exponential function $e^{\lambda t}$ and it plays crucial role in solution of FO state equation. The one parameter Mittag-Leffler function is defined as follows:

Definition 3. *The one parameter Mittag-Leffler function*

$$ E_{\alpha}(x) = \sum_{k=0}^{\infty} \frac{x^k}{\Gamma(k\alpha + 1)}. \tag{3} $$

The two parameter Mittag-Leffler function is defined as follows:

Definition 4. *The two parameters Mittag-Leffler function*

$$ E_{\alpha,\beta}(x) = \sum_{k=0}^{\infty} \frac{x^k}{\Gamma(k\alpha + \beta)}. \tag{4} $$

For $\beta = 1$ the two parameter function (4) turns to one parameter function (3).

The fractional-order, integro-differential operator can be described by various definitions, given by Grünwald and Letnikov (GL Definition), Riemann and Liouville (RL Definition) and Caputo (C Definition). In the further consideration only GL definitions will be used. It is recalled below.

The GL derivative for a function $g(t)$ is defined as follows [25, 26]:

Definition 5. *The Grünwald-Letnikov definition*

$$ {}^{GL}_{0}D^{\alpha}_{t} g(t) = \lim_{h \to 0} h^{-\alpha} \sum_{l=0}^{\left[\frac{t}{h}\right]} (-1)^l \binom{\alpha}{l} g(t - lh). \tag{5} $$

In (5) $0.0 < \alpha \le 1.0$ is the fractional order, h is the sample time, operator [..] is the nearest integer value, $\binom{\alpha}{l}$ is the binomial coefficient:

$$ \binom{\alpha}{l} = \begin{cases} 1, & l = 0 \\ \dfrac{\alpha(\alpha-1)\ldots(\alpha-l+1)}{l!}, & l > 0 \end{cases}. \tag{6} $$

2.2. Elementary FO transfer function

The elementary, scalar, input-output differential equation using elementary fractional operator (1) takes the following form:

$$ T_{\alpha 0} D^{\alpha}_{t} y(t) = -y(t) + u(t), \tag{7} $$

where T_{α} is the quasi time constant, expressed in $[s^{\frac{1}{\alpha}}]$, $u(t)$ is the control signal and $y(t)$ is the output.

The elementary, fractional order transfer function takes the following form:

$$ G_C(s) = \frac{1}{Ts^{\alpha} + 1}. \tag{8} $$

For this transfer function its impulse and step responses are as beneath (see e.g. [25], p. 11):

$$ g_C(s) = \frac{t^{\alpha-1}}{T_{\alpha}} E_{\alpha}\left(-\frac{t^{\alpha}}{T_{\alpha}}\right), \tag{9} $$

$$ y_C(s) = 1(t) - E_{\alpha}\left(-\frac{t^{\alpha}}{T_{\alpha}}\right). \tag{10} $$

In (9) and (10) $E_{\alpha}(\ldots)$ is the one parameter Mittag-Leffler function (3).

2.3. The FOBD approximator and its variable order generalization

The GL definition is the limit case for $h \to 0$ of the FOBD, commonly employed in discrete FO calculations (see e.g. [22], p. 68).

Definition 6. *The Fractional Order Backward Difference*
The Fractional Order Backward Difference is defined as follows:

$$ \Delta^{\alpha} g(k) = \frac{1}{h^{\alpha}} \sum_{l=0}^{L} (-1)^l \binom{\alpha}{l} g(k - lh). \tag{11} $$

In (11) k denotes discrete time moments, h is the sample time, L denotes a memory length necessary to correct approximation of a non-integer order operator. Unfortunately, good accuracy of approximation requires to use a long memory L which can make implementation difficult.

Denote coefficients $(-1)^l \binom{\alpha}{l}$ by d_l:

$$d_l = (-1)^l \binom{\alpha}{l}. \tag{12}$$

The coefficients (12) can be also computed using the following, equivalent, recursive formula given e.g. by [25], p. 12, useful in numerical calculations:

$$\begin{aligned} d_0 &= 1, \\ d_l &= \left(1 - \frac{1+\alpha}{l}\right) d_{l-1}, \quad l = 1, \ldots, L. \end{aligned} \tag{13}$$

In the paper [27] it is proved that:

$$\sum_{l=1}^{\infty} d_l = 1 - \alpha. \tag{14}$$

From (13) and (14) we obtain at once that:

$$\sum_{l=0}^{\infty} d_l = 0. \tag{15}$$

Using (12) the operator (11) can be expressed in shorter form:

$$\Delta^{\alpha} g(t) = \frac{1}{h^{\alpha}} \sum_{l=0}^{L} d_l g(t - lh), \tag{16}$$

and consequently the elementary inertial transfer function (8) employing (16) takes the following form:

$$G(z^{-1}) = \frac{1}{\frac{T}{h^{\alpha}} \sum_{l=0}^{L} d_l z^{-l} + 1}. \tag{17}$$

The characteristic polynomial of the transfer function (17) is a function of sample time h, order α and quasi time constant T. It is as beneath:

$$w(z^{-1}, h, \alpha, T) = \frac{T}{h^{\alpha}} \sum_{l=0}^{L} d_l z^{-l} + 1. \tag{18}$$

The polynomial (18) is the polynomial with constant parameters and its stability is determined by values of its parameters.

Next assume that the order α can take various values from bounded interval $[\underline{\alpha}; \overline{\alpha}]$ at discrete time instants:

$$\alpha = \{\alpha_0, \alpha_1, \ldots, \alpha_L\} \in [\underline{\alpha}; \overline{\alpha}]. \tag{19}$$

The variable order generalization of the operator (11) using various orders (19) has been proposed by [28] and next by [10]. It is employed also in [12] in the form analogical to (16):

$$\Delta^{\alpha} g(t) = \sum_{l=0}^{L} a_l g(t - lh), \tag{20}$$

where:

$$\begin{aligned} a_0 &= \frac{1}{h^{\alpha_0}}, \\ a_l &= \frac{d_{\alpha_l}}{h^{\alpha_l}}, \quad l = 1, \ldots, L. \end{aligned} \tag{21}$$

Coefficients d_{α_l} are computed using recursive formula (13) and variable order α:

$$\begin{aligned} d_{\alpha_0} &= 1, \\ d_{\alpha_l} &= \left(1 - \frac{1+\alpha_l}{l}\right) d_{\alpha_{l-1}}, \quad l = 1, \ldots, L. \end{aligned} \tag{22}$$

The transfer function (8) using (20) takes the following form, analogical to given by [10]:

$$G_{VO}(z^{-1}) = \frac{1}{T \sum_{l=0}^{L} a_l z^{-l} + 1}. \tag{23}$$

Elementary properties of this transfer function are discussed in the paper [10]. Some new results as well as its generalization to VOVT form were proposed in the paper [19] and they are briefly recalled in the next section.

3. The proposed transfer functions

3.1. The $G_{VT}(z^{-1})$ transfer function

Next the time-variant generalization of the transfer function (17) can be proposed. Assume that the order α of the transfer function is constant, but the quasi time constant T can take various values from bounded interval $[\underline{T}; \overline{T}]$ for particular time instants $l = 0, \ldots, L$. This can be written as follows:

$$G_{VT}(z^{-1}) = \frac{1}{h^{-\alpha} \sum_{l=0}^{L} T_l d_l z^{-l} + 1}. \tag{24}$$

where d_l are described by (13) and T_l are bounded values of T for time instants $l = 0, \ldots, L$:

$$T = \{T_0, T_1, \ldots, T_L\} \in [\underline{T}; \overline{T}] \subset \mathbb{R}^+, \quad \underline{T} \le \overline{T}, \quad \overline{T} < \infty. \tag{25}$$

The characteristic polynomial of the transfer function (24) takes the following form:

$$w_{VT}(z^{-1}) = (h^{\alpha} + T_0) + \sum_{l=0}^{L} T_l d_l z^{-l}, \tag{26}$$

where d_l and T_l are expressed by (13) and (25) respectively.

Notice that the approximated transfer function (24) containing the time-variant T_l really is the typical transfer function of order L with various time constants $T_0 - T_L$. Its elementary properties (the step and impulse responses, initial and final values) as well as sufficient stability conditions are given in the paper [19].

3.2. The $G_{VOVT}(z^{-1})$ transfer function

Finally consider the order-variant and time-variant generalization of the transfer function (17). It is described as follows:

$$G_{VOVT}\left(z^{-1}\right) = \frac{1}{\sum_{l=0}^{L} T_l a_l z^{-l} + 1}, \tag{27}$$

where a_l and T_l are described by (21) and (25) respectively.

The transfer function (27) is also the discrete transfer function with constant parameters of order L. Its properties are also analysed with details in the paper [19]. For constant vector α it reduces to transfer function (24) and for both invariant vectors α and T it turns to the discrete transfer function (17).

The characteristic polynomial of the transfer function (27) has the form analogical to (26):

$$w_{VOVT}\left(z^{-1}\right) = (a_0 + T_0) + \sum_{l=0}^{L} T_l a_l z^{-l}, \tag{28}$$

where a_l and T_l are expressed by (21) and (25) respectively.

4. The numerical stability testing

The stability conditions in the paper [19] allow to localise unstable ranges of parameters, but without full and unique information about stability areas.

The interval approach associated with Monte Carlo methods is expected to give better estimation of stability areas for both parameters independently on their variability. To do it define the vertex polynomials of the characteristic polynomial (18). They are as follows:

$$\begin{aligned} w_{ll}(z^{-1},h,\alpha,T) &= w(z^{-1},h,\underline{\alpha},\underline{T}),\\ w_{lh}(z^{-1},h,\alpha,T) &= w(z^{-1},h,\underline{\alpha},\overline{T}),\\ w_{hh}(z^{-1},h,\alpha,T) &= w(z^{-1},h,\overline{\alpha},\overline{T}),\\ w_{hl}(z^{-1},h,\alpha,T) &= w(z^{-1},h,\overline{\alpha},\underline{T}). \end{aligned} \tag{29}$$

The effective numerical algorithm of testing of stability using interval approach and Monte Carlo method is presented beneath.

All experiments were done for memory length $L = 100$. Such a value is often applied during use of FOBD, because it is a compromise between accuracy and numerical complexity of approximation.

The amount of random tests was set equal $N = 1000$. This ensures dense "coverage" of the entire tested area, which increases the probability of the correct result.

The stability test for single set of parameters is tested using simple algorithm 1. It can be applied for each combination of α, T and h, both constant and changing in various ways. It is called in each test.

Algorithm 1. The stability test for single set of parameters
Algorytm 1. Test stabilności dla pojedynczego zestawu parametrów

Input: h, α, T;
Result: *unstab* (it is equal TRUE for unstable polynomial)
1: Compute vector of parameters of polynomial w_{VOVT} using (21)
2: Calculate $unstab := max(abs(roots(w_{VOVT}))) > 1.0$

Stability test for the fixed set $\alpha \times T$ is run using Algorithm 2. Its idea consists in two step stability tests. Firstly all vertices are tested using Algorithm 1 with constant vectors α and T. If there exists unstable vertex, then test is ended with result that the tested area contains unstable sets.

If all vertices are stable, then the whole area $\alpha \times T$ is tested using randomly generated sets of parameters obtained with the use of the MATLAB function *rand*. If all tests are positive, then the whole set $\alpha \times T$ is stable.

Algorithm 2. Stability test for fixed $\alpha \times T$
Algorytm 2. Test stabilności dla ustalonych wektorów $\alpha \times T$

Input: h, α, T, L, N; (N – amount of random tests)
Result: communicate about stability.
1: Check the stability of all vertices polynomials using Algorithm 1.
2:
3: **if** all vertices stable **then**
4: for $n = 1 : N$ **do**
5: $\alpha = \underline{\alpha} + \overline{\alpha} \cdot rand(1, L)$;
6: $T = \underline{T} + \overline{T} \cdot rand(1, L)$;
7: compute *unstab* using Algorithm 1;
8: **if** *unstab* **then**
9: "*Unstable inside*";
10: **return** ;
11: **end if**;
12: **end for**;
13: "*Whole area stable*";
14: **else**
15: "Unstable vertex";
16: **return** ;
17: **end if**

Next some tests of the presented approach need to be presented. The tested parameters and results are collected in the Table 1.

From Table 1 it can be concluded, that the stability of the model is strongly determined by the sample time h. Its shortening leads to instability.

The proposed approach can be also applied to estimate of the stability area. It is done with the use of Algorithm 3. The work of Algorithm 3 will be illustrated with the use of results from Table 1. Its work is illustrated by the Tables 2 and 3. The first one shows the looking for stable ranges of parameters inside initially unstable area, the next one illustrates the expanding of stability ranges.

Algorithm 3. Stability areas $\alpha \times T$ estimation for fixed sample time h
Algorytm 3. Szacowanie obszaru $\alpha \times T$ dla ustalonej wartości okresu próbkowania h

Input: h, α_s, T_s, L, N; (s – starting sets of parameters)
Result: α_f, T_f
1:
2: **if** $\alpha_s \times T_s$ stable **then**
3:
4: **repeat**
5: Expand the range of $\alpha_s \times T_s$;
6: $\alpha_f = \alpha_s$;
7: $T_f = T_s$;
8: **until** $\alpha_f \times T_f$ – stable
9: **else**
10: Narrow the range of $\alpha_s \times T_s$;
11:
12: if $\alpha_s \times T_s$ – stable **then**
13:
14: **end if**
15: **end if**

Table 1. Stability tests for various sets of parameters. For each experiment: L = 100, N = 1000
Tabela 1. Testy stabilności dla różnych zestawów parametrów. W każdym eksperymencie: L = 100, N = 1000

No.	h	T	α	stable?
1	0.1	[0.9;1.1]	[0.45;0.55]	Yes
2	0.5	[0.9;1.1]	[0.45;0.55]	Yes
3	0.01	[0.9;1.1]	[0.45;0.55]	No, unstable inside
4	0.5	[0.7;1.5]	[0.75;0.85]	Yes
5	0.1	[0.7;1.5]	[0.75;0.85]	No, unstable vertices $_{lh}$, $_{hh}$
6	0.01	[0.7;1.5]	[0.75;0.85]	No, unstable vertices $_{lh}$, $_{hh}$
7	0.1	[1.0;2.0]	[0.15;0.25]	Yes
8	0.5	[1.0;2.0]	[0.15;0.25]	Yes
9	0.01	[1.0;2.0]	[0.15;0.25]	No, unstable vertex $_{hh}$

Table 2. Estimation of stability area for initially unstable parameters. For each experiment: L = 100, N = 1000
Tabela 2. Oszacowanie obszaru stabilności dla początkowo niestabilnych parametrów. W każdym eksperymencie: L = 100, N = 1000

No. of exp. (Table 1)	h	T_s	α_s	T_f	α_f
3	0.01	[0.9;1.1]	[0.45;0.55]	[0.9;1.0]	[0.48;0.52]
5	0.1	[0.7;1.5]	[0.75;0.85]	[0.78;1.1]	[0.75;0.85]
6	0.01	[0.7;1.5]	[0.75;0.85]	[0.78;1.1]	[0.75;0.85]
9	0.01	[1.0;2.0]	[0.15;0.25]	[1.0;1.5]	[0.15;0.25]

Table 3. Expanding of stability area for stable parameters. For each experiment: L = 100, N = 1000
Tabela 3. Rozszerzenie obszaru stabilności dla stabilnych parametrów. W każdym eksperymencie: L = 100, N = 1000

No. of exp. (Table 1)	h	T_s	α_s	T_f	α_f
1	0.1	[0.9;1.1]	[0.45;0.55]	[0.3;1.1]	[0.45;0.60]
2	0.5	[0.9;1.1]	[0.45;0.55]	[0.3;1.6]	[0.35;0.60]
7	0.1	[1.0;2.0]	[0.15;0.25]	[0.2;2.1]	[0.2;2.1]
8	0.5	[1.0;2.0]	[0.15;0.25]	[0.2;2.2]	[0.1;0.3]

5. Final Conclusions

The main final conclusion from this paper is that the use of the Monte Carlo method associated to interval approach allows to test the stability of the considered, variable parameters fractional transfer function. The properly estimated range of tests uniquely points the areas of stability.

In addition, starting tests from vertices of examined area allows to quickly detect unstability. This significantly speeds a test.

Further investigations of the considered issue cover the looking for "stronger", analytical conditions of the stability as well as the use of the proposed VOVT transfer function in modeling and control.

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Numeryczna analiza stabilności transmitancji ułamkowego rzędu o zmiennych parametrach

Streszczenie: W artykule zaprezentowano numeryczną analizę praktycznej stabilności nowych modeli transmitancyjnych niecałkowitego rzędu o zmiennych parametrach. Rozważane transmitancje są budowane z wykorzystaniem aproksymacji FOBD o zmiennych parametrach. Postać transmitancji implikuje możliwość podania tylko dostatecznych warunków niestabilności lub równoważnie koniecznych warunków stabilności. To nie zapewnia pełnej i jednoznacznej informacji o stabilności. Proponowany algorytm numeryczny bazuje na podejściu przedziałowym i metodzie Monte Carlo. Podejście przedziałowe pozwala na szybkie wykrycie niestabilności na wierzchołkach testowanego obszaru. Proponowany algorytm pozwala na sprawdzenie stabilności w zadanym zakresie oraz na oszacowanie obszarów stabilności na płaszczyźnie parametrów modelu.

Słowa kluczowe: transmitancja ułamkowego rzędu, system niestacjonarny, aproksymacja FOBD, stabilność, metoda Monte Carlo

Prof. Krzysztof Oprzędkiewicz, PhD DSc
kop@agh.edu.pl
ORCID: 0000-0002-8162-0011

He was born in Krakow in 1964. He obtained MSc in electronics in 1988, PhD and DSc in Automatics and Robotics in 1995 and 2009 at AGH University of Science and Technology (Krakow, Poland). He has been working at AGH University in Department of Automatic Control since 1988, recently as the full professor. In 2012–2016 he was a deputy dean of faculty of Electrical Engineering, Automatic Control, Computer Science and Biomedical Engineering at AGH University. Recently he is the head of the Department of Automatic Control and Robotics at AGH University. Since 2000 he has been working also at the University of Applied Sciences in Tarnow, recently as the full professor at the Faculty of Technical Sciences. Since 2020 he is also a member of Committee on Automatic Control and Robotics of the Polish Academy of Sciences, since 2024 he is also the chairman of the Teaching Section in this Committee. His research covers infinite dimensional systems, fractional order modeling and control, uncertain parameter systems, positive systems, industrial automation, PLC and SCADA systems..